Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 1

Question:

A train decelerates uniformly from 35 m s $^{-1}$ to 21 m s $^{-1}$ in a distance of 350 m. Calculate

a the deceleration,

b the total time taken, under this deceleration, to come to rest from a speed of 35 m s $^{-1}$.

Solution:

a $u = 35, v = 21, s = 350, a = ?$

$v^2 = u^2 + 2as$

$21^2 = 35^2 + 2 \times s \times 350$

$a = \frac{21^2 - 35^2}{700} = -1.12$

The deceleration of the train is 1.12 m s^{-2}.

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

b $u = 35, v = 0, a = -1.12, t = ?$

$v = u + at$

$0 = 35 - 1.12t$

$t = \frac{35}{1.12} = 31.25$

The total time taken to come to rest is 31.25 s.

You use the value of a you found in part **a** in part **b**. As the train is decelerating, a is negative.

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Review Exercise
Exercise A, Question 2

Question:

In taking off, an aircraft moves on a straight runway *AB* of length 1.2 km. The aircraft moves from *A* with initial speed 2 m s^{-1}. It moves with constant acceleration and 20s later it leaves the runway at *C* with speed 74 m s^{-1}. Find

a the acceleration of the aircraft,

b the distance *BC*.

Solution:

a $u = 2, t = 20, v = 74, a = ?$

$v = u + at$

$74 = 2 + 20a$

$a = \frac{74-2}{20} = 3.6$

The acceleration of the aircraft is 3.6 m s^{-2}.

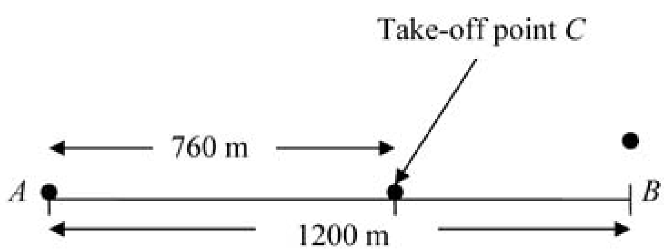
There is no s here, so you choose the formula without s, $v = u + at$.

b $u = 2, t = 20, v = 74, s = ?$

$s = \left(\frac{u+v}{2}\right)t$

$= \left(\frac{2+74}{2}\right) \times 20 = 760$

You could use the value of a you found in part **a** but, if it can be avoided without causing a lot of extra work, it is safer not to use your answer from **a**. Everyone makes mistakes from time to time!



The distance AC is 760 m.

The distance BC is $(1200 - 760) \text{ m} = 440 \text{ m}$.

The aircraft takes off 760 m from its starting point A. The runway is 1.2 km, which is 1200 m, long. So the aircraft is $(1200 - 760)\text{ m}$ from the end C of the runway.

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Review Exercise
Exercise A, Question 3

Question:

A car is moving along a straight horizontal road at a constant speed of 18 m s^{-1}. At the instant when the car passes a lay-by, a motorcyclist leaves the lay-by, starting from rest, and moves with constant acceleration 2.5 m s^{-2} in pursuit of the car. Given that the motorcyclist overtakes the car *T* seconds after leaving the lay-by, calculate

a the value of *T*,

b the speed of the motorcyclist at the instant of passing the car.

Solution:

a After time t, let the distance moved by the car be s_1 and the distance moved by the motor cycle s_2.

The distance moved by the car is given by

$$s_1 = 18t$$

The car is travelling at a constant speed, so you use distance = speed × time to obtain an expression for the distance moved by the car.

The distance moved by the motor cycle is given by

$$s = ut + \frac{1}{2}at^2, \text{ with } u = 0 \text{ and } a = 2.5$$

$$s_2 = 0 \times t + \frac{1}{2} \times 2.5t^2 = 1.25t^2$$

When $t = T$, $s_1 = s_2$

$$18T = 1.25T^2$$

As the car and the motor cycle were level at the lay-by, when the motor cycle overtakes the car, they have travelled the same distance. Equating s_1 to s_2 gives and equation you can solve.

$$1.25T^2 - 18T = T(1.25T - 18) = 0$$

$T = 0$ is a solution of this equation but that is the time at the lay-by and can be ignored.

$$T = \frac{18}{1.25} = 14.4$$

b $u = 0, a = 2.5, t = 14.4, v = ?$

$$v = u + at$$
$$= 0 + 2.5 \times 14.4 = 36$$

The speed of the motor cycle at the instant of passing the car is 36 m s^{-1}.

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Review Exercise
Exercise A, Question 4

Question:

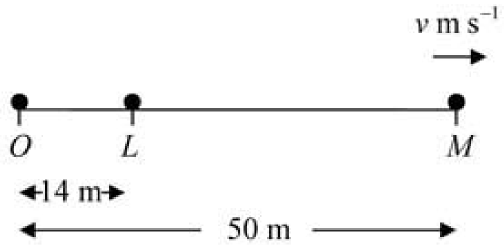
A particle moves with constant acceleration along the straight line *OLM* and passes through the points *O*, *L* and *M* at times 0 s, 4 s and 10 s respectively. Given that *OL* = 14 m and *OM* = 50 m, find

a the acceleration of the particle,

b the speed of the particle at *M*.

Solution:

a



You are asked to find two variables in this question, a and v. You are given information about two variables, t and s. You are neither given any information nor asked about the initial speed u. The most efficient way of solving the question is to use the formula without u, $s = vt - \frac{1}{2}at^2$. This enables you to solve both parts of the question together.

From L to M
The particle takes $(10-4)\text{s} = 6\text{ s}$ to travel a distance $(50-14)\text{m} = 36\text{ m}$.

$t = 6,\ s = 36,\ a = ?,\ v = ?$

$$s = vt - \frac{1}{2}at^2$$

$$36 = 6v - \frac{1}{2}a \times 6^2 \Rightarrow 6v - 18a = 36$$

$$\Rightarrow v - 3a = 6 \ldots \ldots (1)$$

You obtain two linear simultaneous equations from the data. You solve these in the way you learnt for GCSE.

From O to M, $t = 10,\ s = 50,\ a = ?,\ v = ?$

$$s = vt - \frac{1}{2}at^2$$

$$50 = 10v - \frac{1}{2}a \times 10^2 \Rightarrow 10v - 50a = 50$$

$$\Rightarrow v - 5a = 5 \ldots\ (2)$$

(1) − (2)

$$2a = 1 \Rightarrow a = 0.5$$

The acceleration of the particle is 0.5 m s^{-2}.

b Substitute $a = 0.5$ into (1)

$$v - 1.5 = 6 \Rightarrow v = 7.5$$

The speed of the particle at M is 7.5 m s^{-1}.

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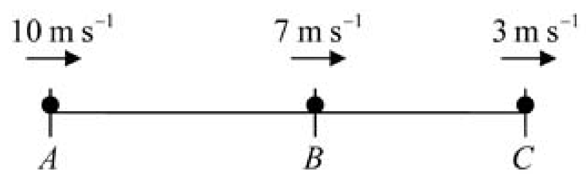
Review Exercise
Exercise A, Question 5

Question:

A particle P moves in a straight line with constant retardation. At the instants when P passes through the points A, B and C, it is moving with speeds 10 m s^{-1}, 7 m s^{-1} and 3 m s^{-1} respectively.

Show that $\frac{AB}{BC} = \frac{51}{40}$.

Solution:



From A to B

$u = 10,\ v = 7,\ s = AB$

$v^2 = u^2 + 2as$

$7^2 = 10^2 + 2a \times AB$

$2a \times AB = 49 - 100 = -51 \quad \ldots\ \ldots\ (1)$

You are given information about speeds and asked about distances. Time will not come into the question, so it is reasonable to try and make two equations with $v^2 = u^2 + 2as$.

From B to C

$u = 7,\ v = 3,\ s = BC$

$v^2 = u^2 + 2as$

$3^2 = 7^2 + 2a \times BC$

$2a \times BC = 9 - 49 = -40 \quad \ldots\ \ldots\ (2)$

Dividing equation (1) by equation (2), $2a$ is in both the numerator and denominator of the fraction and so can be "cancelled".

$(1) \div (2)$

$$\frac{\cancel{2a} \times AB}{\cancel{2a} \times BC} = \frac{-51}{-40} = \frac{51}{40}$$

$$\frac{AB}{BC} = \frac{51}{40}, \text{ as required.}$$

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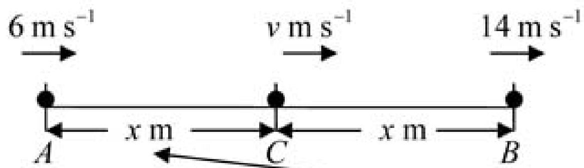
Review Exercise
Exercise A, Question 6

Question:

A car moving with uniform acceleration along a straight level road, passed points A and B when moving with speed 6 m s^{-1} and 14 m s^{-1} respectively. Find the speed of the car at the instant that it passed C, the mid-point of AB.

Solution:



You do not know the distances but as C is the mid-point of AB if you let AC be x, then $AB = 2x$. This use of x as the unknown allows you to form equations.

From A to B

$u = 6,\ v = 14,\ s = AB = 2x$

$v^2 = u^2 + 2as$

$14^2 = 6^2 + 2a \times 2x \Rightarrow 196 = 36 + 4ax$

$ax = \dfrac{196 - 36}{4} = 40$

You are given information about speeds and distances.
Time will not come into the question, so it is reasonable to try and make two equations with $v^2 = u^2 + 2as$.

From A to C

$u = 6,\ s = AC = x,\ v = ?$

$v^2 = u^2 + 2as$

$\quad = 6^2 + 2ax = 36 + 2 \times 40 = 116$

$v = \sqrt{116} \approx 10.8$

The speed of the car at C is 10.8 m s^{-1} (3 s.f.).

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Review Exercise
Exercise A, Question 7

Question:

A particle P is moving along the x-axis with constant deceleration 3 m s^{-2}. At time t = 0 s, P is passing through the origin O and is moving with speed u m s^{-1} in the direction of x increasing. At time t = 8 s, P is instantaneously at rest at the point A. Find

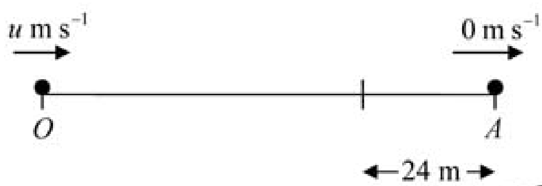
a the value of u,

b the distance OA,

c the times at which P is 24 m from A.

Solution:

a



'Instantaneously at rest' is translated into algebra as $v = 0$.

$a = -3, t = 8, v = 0, u = ?$
$v = u + at$
$0 = u - 3 \times 8 \Rightarrow u = 24$

b $a = -3, t = 8, v = 0, s = ?$

$s = vt - \frac{1}{2}at^2$

$= 0 \times 8 - \frac{1}{2} \times (-3) \times 8^2 = 96$

$OA = 96$ m

You do not need u here, since you can use the formula without u, $s = vt - \frac{1}{2}at^2$.

c 24 m from A is $(96 - 24)$m $= 72$ m from O

$s = 72, a = -3, u = 24, t = ?$

$s = ut + \frac{1}{2}at^2$

$72 = 24t + \frac{1}{2} \times (-3) \times t^2 = 24t - 1.5t^2$

$3t^2 - 48t + 144 = 0$

Multiply the equation by 2 and rearrange.

Dividing throughout by 3

$t^2 - 16t + 48 = (t-4)(t-12) = 0$

$t = 4, 12$

P is 24 m for A at times $t = 4$ s and $t = 12$ s.

You may use any method to solve the quadratic equation but if you did use the formula you would be expected to obtain exact answers.

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Review Exercise
Exercise A, Question 8

Question:

A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points *A*, *B* and *C*, where $AB = 50$ m and $BC = 50$ m. The front of the train passes *A* with speed 22.5 m s^{-1}, and 2 s later it passes *B*. Find

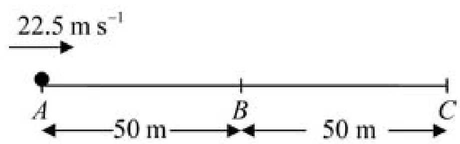
a the acceleration of the train,

b the speed of the front of the train when it passes *C*,

c the time that elapses from the instant the front of the train passes *B* to the instant it passes *C*.

Solution:

a



From A to B

$u = 22.5,\ s = 50,\ t = 2,\ a = ?$

$s = ut + \frac{1}{2}at^2$

There is no v here, so you use the formula without v, $s = ut + \frac{1}{2}at^2$.

$50 = 22.5 \times 2 + \frac{1}{2}a \times 2^2 = 45 + 2a$

$a = \frac{50 - 45}{2} = 2.5$

The acceleration of the train is $2.5\ \text{m s}^{-2}$.

b From A to C

$u = 22.5,\ s = 100,\ a = 2.5,\ v = ?$

$v^2 = u^2 + 2as$

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

$\quad = 22.5^2 + 2 \times 2.5 \times 100 = 1006.25$

$v = \sqrt{1006.25} = 31.7\ \ldots$

The speed at C is $31.7\ \text{m s}^{-1}$ (3 s.f.).

Where no accuracy is specified, it is reasonable to give your answer to 3 significant figures.

c To find time from A to C

$u = 22.5,\ v = \sqrt{1006.25},\ a = 2.5,\ t = ?$

$v = u + at$

$\sqrt{1006.25} = 22.5 + 2.5t$

$t = \frac{\sqrt{1006.25} - 22.5}{2.5} = 3.688\ \ldots$

The time from B to C is

$(3.69 - 2)\text{s} = 1.69\ \text{s}$ (3 s.f.).

It is possible to find the time directly using $s = vt - \frac{1}{2}at^2$ but this would give an awkward quadratic with difficult coefficients. $v = u + at$ is easy to use and, in this case, it is easy to adjust the time found, from A to C, by subtracting the time taken to go from A to B. This gives the time from B to C as required.

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Review Exercise
Exercise A, Question 9

Question:

A particle X, moving along a straight line with constant speed 4 m s $^{-1}$, passes through a fixed point O. Two seconds later, another particle Y, moving along the same straight line and in the same direction, passes through O with speed 6 m s $^{-1}$. The particle Y is moving with constant deceleration 2 m s $^{-2}$.

a Write down expressions for the velocity and displacement of each particle t seconds after Y passed through O.

b Find the shortest distance between the particles after they have both passed through O.

c Find the value of t when the distance between the particles has increased to 23 m.

Solution:

a For X, let the velocity at time t second be v_x m s^{-1} and the displacement s_x m .

$$v_x = 4$$
$$s_x = 4(t+2)$$

t seconds is the time since Y passed through O. X passed through O 2 seconds earlier, so it is $(t+2)$s since X passed through O.
As X is moving with constant speed, distance = speed × time .

For Y, let the velocity at time t second be v_y m s^{-1} and the displacement s_y m .

$u = 6, a = -2$

$$v_y = u + at$$
$$= 6 - 2t$$
$$s_y = ut + \frac{1}{2}at^2$$
$$= 6t - t^2$$

b The distance between X and Y, d say, is given by

$$d = s_x - s_y = 4(t+2) - (6t - t^2)$$
$$= 8 - 2t + t^2$$
$$= 7 + 1 - 2t + t^2$$
$$= 7 + (t-1)^2 \geqslant 7$$

The shortest distance is 7 m.

All real numbers squared are non-negative.
So $(t-1)^2 \geqslant 0$ and it follows that
$7 + (t-1)^2 \geqslant 7 + 0 = 7$.
There are alternative solutions using differentiation.

c

$$7 + (t-1)^2 = 23$$
$$(t-1)^2 = 16$$
$$t - 1 = 4$$
$$t = 5$$

The negative square root of 16, -4, gives $t = -3$. This can be ignored as the time would be before Y passed through O.

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Review Exercise
Exercise A, Question 10

Question:

A stone is projected vertically upwards from a point A with initial speed u m s^{-1}. It takes 3.5 s to reach its maximum height above A. Find

a the value of u,

b the maximum height of the stone above A.

Solution:

a Taking the upwards direction as positive.

At the greatest height the stone is instantaneously at rest. In algebra, $v = 0$.

$v = 0, t = 3.5, a = -9.8, u = ?$
$v = u + at$
$0 = u - 9.8 \times 3.5$
$u = 34.3$

$u = 34$ (2 s.f.)

When you model an object moving in a vertically as a particle moving with constant acceleration of magnitude 9.8 m s^{-2}, you should give your answers corrected to 2 significant figures. You should, however, show your working to at least 3 significant figures.

b $v = 0, t = 3.5, a = -9.8, s = ?$

$s = vt - \frac{1}{2}at^2$

$= 0 - \frac{1}{2} \times (-9.8) \times 3.5^2 = 60.025$

The maximum height above A is 60 m (2 s.f.).

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Review Exercise
Exercise A, Question 11

Question:

A small ball is projected vertically upwards from a point *A*. The greatest height reached by the ball is 40 m above *A*. Calculate

a the speed of projection,

b the time between the instant that the ball is projected and the instant it returns to *A*.

Solution:

a Taking the upwards direction as positive.

$s = 40, v = 0, a = -9.8, u = ?$

At the greatest height the velocity of the ball is instantaneously zero.

$v^2 = u^2 + 2as$

$0^2 = u^2 - 2 \times 9.8 \times 40$

$u^2 = 784 \Rightarrow u = 28 \ldots$

The speed of projection is 28 m s^{-1}.

When the ball returns to A, its displacement from A is zero.

b $s = 0, u = 28, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 28t - 4.9t^2 = t(28 - 4.9t)$

$t = 0, t = \frac{28}{4.9} = 5.714 \ldots$

The solution $t = 0$ represents the time of projection. You choose the other solution for the time of return.

The time taken to return to A is 5.7 s, (2 s.f.).

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Review Exercise
Exercise A, Question 12

Question:

A ball is projected vertically upwards and takes 3 seconds to reach its highest point. At time t seconds, the ball is 39.2 m above its point of projection. Find the possible values of t.

Solution:

Find the speed of projection.
Taking the upwards direction as positive.
$v = 0, t = 3, a = -9.8, u = ?$

$v = u + at$
$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

There are two values of t, one as the ball ascends and one as it descends. To find two values of t, you will need to form a quadratic equation using $s = ut + \frac{1}{2}at^2$.
Before you can do this, you will need to use the information, that the ball reaches its greatest height in 3 s, to find u.

$s = 39.2, u = 29.4, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$39.2 = 29.4t - 4.9t^2$

$4.9t^2 - 29.4t + 39.2 = 0$

You may solve the quadratic by any exact method. Dividing all the terms by 4.9 gives a quadratic equation with integer coefficients, this equation factorises easily.

Dividing all terms by 4.9

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

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Review Exercise
Exercise A, Question 13

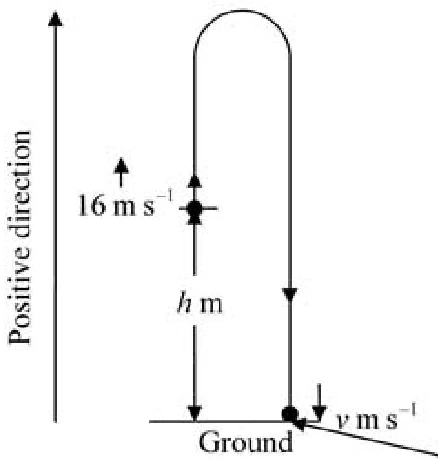
Question:

A stone is thrown vertically upwards with speed 16 m s^{-1} from a point h metres above the ground. The stone hits the ground 4 s later. Find

a the value of h,

b the speed of the stone as it hits the ground.

Solution:



When the stone hits the ground, it is h metres **below** the point of projection and so the displacement, s, is "minus h".

a $u = 16, a = -9.8, t = 4, s = -h$

$$s = ut + \frac{1}{2}at^2$$
$$-h = 16 \times 4 - 4.9 \times 4^2$$
$$= -14.4$$
$$h = 14 \text{ (2 s.f.)}.$$

b $u = 16, a = -9.8, t = 4, v = ?$

$$v = u + at$$
$$= 16 - 9.8 \times 4 = -23.2$$

The speed of the stone as it hits the ground is 23 m s^{-1} (2 s.f.).

The velocity, $v \text{ m s}^{-1}$, is -23.2 m s^{-1} because when the stone hits the ground it is moving downwards and the positive direction has been taken upwards. However, you have been asked for the speed of the stone as it hits the ground and speed is the magnitude of the velocity, 23 m s^{-1} (2 s.f.).

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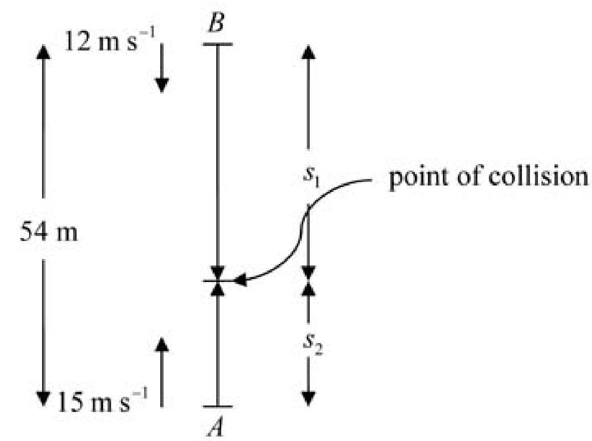
Review Exercise
Exercise A, Question 14

Question:

Two balls are projected simultaneously from two points A and B. The point A is 54 m vertically below B. Initially one ball is projected from A towards B with speed 15 m s^{-1}. At the same time the other ball is projected from B towards A with speed 12 m s^{-1}.

Find the distance between A and the point where the balls collide.

Solution:



From B, take the downwards direction as positive
$u = 12, a = 9.8$

$$s_1 = ut + \frac{1}{2}at^2$$
$$= 12t + 4.9t^2 \quad \dots \dots (1)$$

From A, take the upwards direction as positive
$u = 15, a = -9.8$

$$s_2 = ut + \frac{1}{2}at^2$$
$$= 15t - 4.9t^2 \quad \dots \dots (2)$$

You can form an equation in t using the relation that, at the point of collision, the displacement downwards of the ball projected from B added to the displacement upwards of the ball projected from A is 54 m, the distance between A and B.

(1) + (2)
$$s_1 + s_2 = 12t + 4.9t^2 + 15t - 4.9t^2$$
$$54 = 27t$$
$$t = 2$$

Substitute $t = 2$ into (2)
$$s_2 = 15 \times 2 - 4.9 \times 2^2 = 10.4$$

The balls collide at a point 10 m (2 s.f.) above A.

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Review Exercise
Exercise A, Question 15

Question:

A particle is projected vertically upwards from a point *A* with speed u m s^{-1}. The particle takes $2\frac{6}{7}$ s to reach its greatest height above *A*. Find

a the value of u,

b the total time for which the particle is more than $17\frac{1}{2}$ m above *A*.

Solution:

a Taking upwards as the positive direction

$v = 0, a = -9.8, t = 2\frac{6}{7} = \frac{20}{7}, u = ?$

$v = u + at$

$0 = u - 9.8 \times \frac{20}{7}$

$u = 9.8 \times \frac{20}{7} = 28$

b $s = 17.5, u = 28, a = -9.8, t = ?$

$s = ut + \frac{1}{2}at^2$

$17.5 = 28t - 4.9t^2$

$4.9t^2 - 28t + 17.5 = 0$

Divide all terms by 0.7

$7t^2 - 40t + 25 = 0$

$(7t - 5)(t - 5) = 0$

$t = \frac{5}{7}, 5$

The time for which the particle is more than $17\frac{1}{2}$ m above *A* is $\left(5 - \frac{5}{7}\right)$s $= 4\frac{2}{7}$ s.

The particle passes through the point $17\frac{1}{2}$ m above *A* twice. The first time is as it ascends. The second, as it descends. The first stage of the solution to part (b) is to find the two values of t when these happen.

The difference between these times is the total time for which the particle is more than $17\frac{1}{2}$ m above *A*.

This is an exact answer and can be left as it is. 4.3 s would also be an acceptable answer.

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Review Exercise
Exercise A, Question 16

Question:

A particle moves along a horizontal straight line. The particle starts from rest, accelerates at 2 m s $^{-2}$ for 3 seconds, and then decelerates at a constant rate coming to rest in a further 8 seconds.

a Sketch a speed–time graph to illustrate the motion of the particle.

b Find the total distance travelled by the particle during these 11 seconds.

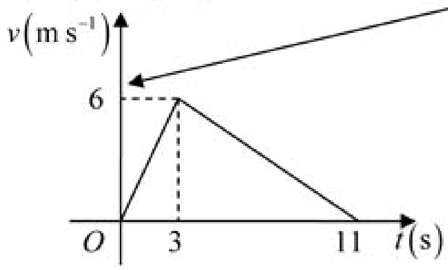
Solution:

a To find the velocity after 3 seconds

$u = 0, a = 2, t = 3, v = ?$

$v = u + at$

$v = 0 + 2 \times 3 = 6$



You need to find the speed after 3 seconds, both to draw the graph in part (a) and to calculate the area of the triangle in part (b). You can do this either by using $v = u + at$, as is shown here, or by using the key point that the gradient of a speed-time graph represents the acceleration.

b Area $= \frac{1}{2}$base $\times$ height

$s = \frac{1}{2} 11 \times 6 = 33$

The area of the triangle between the speed-time graph and the axis represents the distance travelled.

The total distance travelled by the particle is 33 m.

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Review Exercise
Exercise A, Question 17

Question:

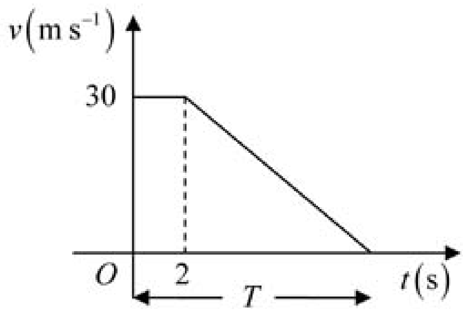
A man is driving a car on a straight horizontal road. He sees a junction *S* ahead, at which he must stop. When the car is at the point *P*, 300 m from *S*, its speed is 30 m s^{-1}. The car continues at this constant speed for 2 s after passing *P*. The man then applies the brakes so that the car has constant deceleration and comes to rest at *S*.

a Sketch a speed-time graph to illustrate the motion of the car in moving from *P* to *S*.

b Find the time taken by the car to travel from *P* to *S*.

Solution:

a



b Let T seconds be the time taken to travel from A to B.

$$s = \frac{1}{2}(a+b)h$$

The area of the trapezium between the speed-time graph and the axis represents the distance travelled.

$$300 = \frac{1}{2}(2+T)\times 30$$
$$= 15(2+T) = 30 + 15T$$
$$T = \frac{300-30}{15} = 18$$

You know that the distance is 300 m. The shorter of the parallel sides of the trapezium represents 2 s and the longer side T seconds. You then form an equation using the formula for the area and solve it for T.

The car takes 18 s to travel from A to B.

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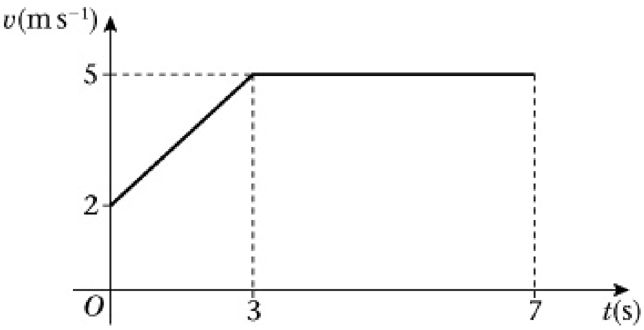
Review Exercise
Exercise A, Question 18

Question:



The figure shows the speed–time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from t = 0 to t = 3, and from t = 3 to t = 7, are straight lines. The section from t = 3 to t = 7 is parallel to the t-axis.

State what can be deduced about the motion of the cyclist from the fact that

a the graph from t = 0 to t = 3 is a straight line,

b the graph from t = 3 to t = 7 is parallel to the t-axis.

c Find the distance travelled by the cyclist during this 7 s period.

Solution:

a For the first 3 s the cyclist is moving with constant acceleration.

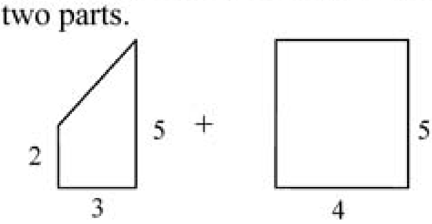
b For the remaining 4 s the cyclist is moving with constant speed.

c area = trapezium + rectangle

$$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$$
$$= 10.5 + 20 = 30.5$$

The distance travelled by the cyclist is 30.5 m.

The area, representing the distance travelled, is made up of two parts.



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Review Exercise
Exercise A, Question 19

Question:

A train stops at two stations 7.5 km apart. Between the stations it takes 75 s to accelerate uniformly to a speed 24 m s^{-1}, then travels at this speed for a time T seconds before decelerating uniformly for the final 0.6 km.

a Draw a speed–time graph to illustrate this journey.

Hence, or otherwise, find

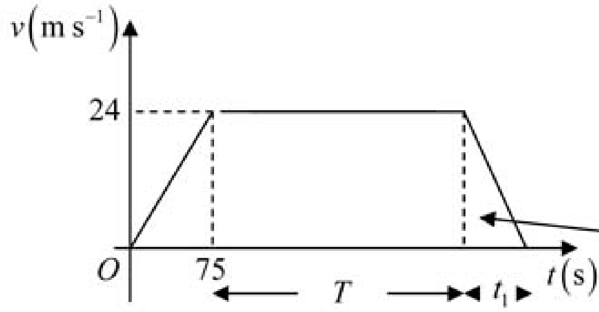
b the deceleration of the train during the final 0.6 km,

c the value of T,

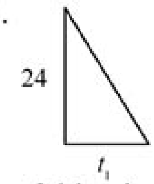
d the total time for the journey.

Solution:

a



The motion of the train while decelerating is represented by this triangle .



The area of this triangle represents the distance travelled while decelerating, which is given to be 0.6 km = 600m.

b Let time for which the train decelerates be t_1 s .

While decelerating

$$\text{area} = \frac{1}{2}\text{base} \times \text{height}$$

$$600 = \frac{1}{2}t_1 \times 24$$

$$t_1 = \frac{1200}{24} = 50$$

Acceleration is represented by the gradient.

$$a = \frac{-24}{t_1} = -\frac{24}{50} = -0.48$$

The deceleration is 0.48 m s^{-2}.

The shorter of the parallel sides of the trapezium represents T seconds.
The longer of the parallel sides of the trapezium represents

$$(75 + T + t_1)\text{s} = (75 + T + 50)\text{s}$$
$$= (T + 125)\text{s}$$

c For the whole journey

$$s = \frac{1}{2}(a + b)h$$

$$7500 = \frac{1}{2}(T + T + 125) \times 24$$
$$= 12(2T + 125) = 24T + 1500$$

$$T = \frac{7500 - 1500}{24} = 250$$

d Total time is $(75 + T + t_1)\text{s} = (75 + 250 + 50)\text{s} = 375 \text{ s}$.

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Review Exercise
Exercise A, Question 20

Question:

A car accelerates uniformly from rest to a speed of 20 m s $^{-1}$ in T seconds. The car then travels at a constant speed of 20 m s $^{-1}$ for $4T$ seconds and finally decelerates uniformly to rest in a further 50 s.

a Sketch a speed–time graph to show the motion of the car.

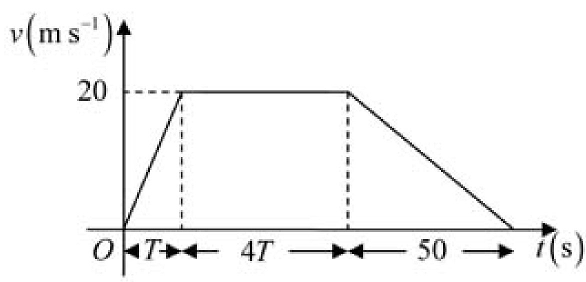
The total distance travelled by the car is 1220 m. Find

b the value of T,

c the initial acceleration of the car.

Solution:

a



b $s = \frac{1}{2}(a+b)h$

$1220 = \frac{1}{2}(4T + 5T + 50) \times 20$

$= 10(9T + 50) = 90T + 500$

$T = \frac{1220 - 500}{90} = 8$

The shorter of the parallel sides of the trapezium represents $4T$ seconds.
The longer of the parallel sides of the trapezium represents $(T + 4T + 50)\text{s} = (5T + 50)\text{s}$.

c $a = \frac{20}{T} = \frac{20}{8} = 2.5$

The initial acceleration of the car is 2.5 m s^{-2}.

The gradient of the line for the first T seconds represents the initial acceleration.
The gradient is $\frac{\text{the increase in velocity}}{\text{the increase in time}}$.

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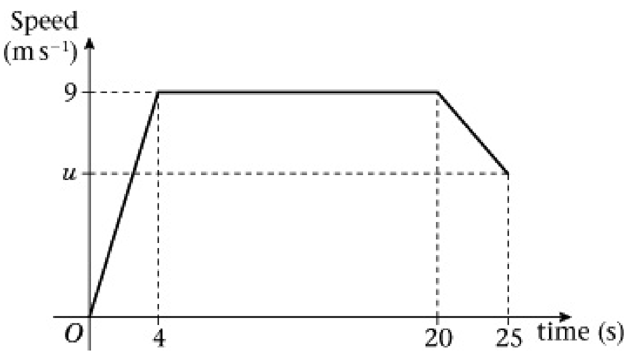
Review Exercise
Exercise A, Question 21

Question:



A sprinter runs a race of 200 m. Her total time for running the race is 25 s. The figure is a sketch of the speed-time graph for the motion of the sprinter. She starts from rest and accelerates uniformly to a speed of 9 m s^{-1} in 4 s. The speed of 9 m s^{-1} is maintained for 16 s and she then decelerates uniformly to a speed of u m s^{-1} at the end of the race. Calculate

a the distance covered by the sprinter in the first 20 s of the race,

b the value of u,

c the deceleration of the sprinter in the last 5 s of the race.

Solution:

a For first 20 s

$$s=\frac{1}{2}(a+b)h$$

$$=\frac{1}{2}(16+20)\times 9=162$$

The distance covered in the first 20 s is 162 m.

b The distance covered in the last 5 s is

$(200-162)\text{m}=38\text{ m}$.

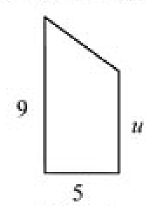
$$s=\frac{1}{2}(a+b)h$$

$$38=\frac{1}{2}(u+9)\times 5$$

$$u+9=\frac{38\times 2}{5}=15.2$$

$$u=6.2$$

The trapezium representing the distance travelled in the last 5 s has shape



The area of this trapezium must equal 38 and you can form an equation in u.

c $$a=\frac{u-9}{5}=\frac{6.2-9}{5}=\frac{-2.8}{5}=-0.56$$

The deceleration in the last 5 s is 0.56 m s^{-2}.

The gradient of the line representing the last 5 s of motion is negative. v decreases by 2.8 as t increases by 5.

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Review Exercise
Exercise A, Question 22

Question:

An electric train starts from rest at a station *A* and moves along a straight level track. The train accelerates uniformly at 0.4 m s^{-2} to a speed of 16 m s^{-1}. The speed is then maintained for a distance of 2000 m. Finally the train retards uniformly for 20 s before coming to rest at a station *B*. For this journey from *A* to *B*,

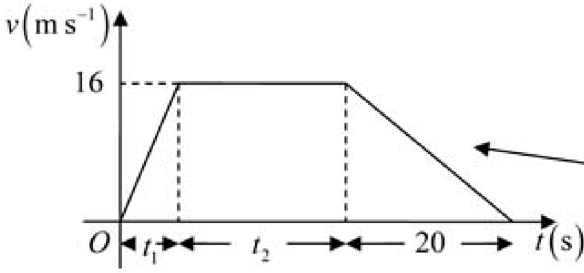
a find the total time taken,

b find the distance from *A* to *B*,

c sketch the *distance*–time graph, showing clearly the shape of the graph for each stage of the journey.

Solution:

a



You were not asked to sketch a speed-time graph but it helps you to visualise the problem and you will be able to work out part **b** using the area of the trapezium.

Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

During acceleration

$v = u + at$

$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed

$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

The total time is $(t_1 + t_2 + 20)\text{s} = (40 + 125 + 20)\text{s} = 185\text{ s}$.

At constant speed, distance = speed × time.

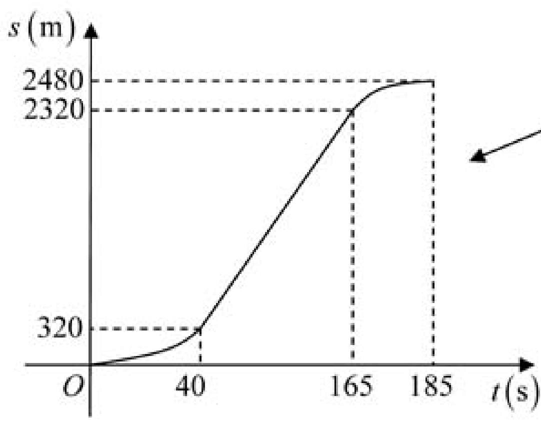
b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(125 + 185) \times 16 = 2480$

$AB = 2480$ m

c The distance moved in the first 40 s of motion is given by

$s = \frac{1}{2} \times 40 \times 16 = 320$



A distance-time graph is a straight line when the object is moving at a constant speed and a curve when the object is accelerating or decelerating.

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Review Exercise
Exercise A, Question 23

Question:

A car starts from rest at a point *S* on straight racetrack. The car moves with constant acceleration for 20 s, reaching a speed of 25 m s^{-1}. The car then travels at a constant speed of 25 m s^{-1} for 120 s. Finally it moves with constant deceleration, coming to rest at a point *F*.

a Sketch a speed–time graph to illustrate the motion of the car.

The distance between *S* and *F* is 4 km.

b Calculate the total time the car takes to travel from *S* to *F*.

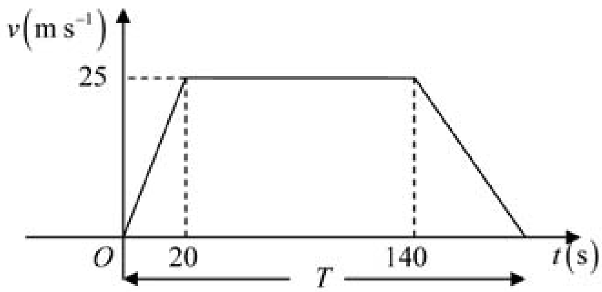
A motorcycle starts at *S*, 10 s after the car has left *S*. The motorcycle moves with constant acceleration from rest and passes the car at a point *P* which is 1.5 km from *S*. When the motorcycle passes the car, the motorcycle is still accelerating and the car is moving at a constant speed. Calculate

c the time the motorcycle takes to travel from *S* to *P*,

d the speed of the motorcycle at *P*.

Solution:

a



b Let the total time be T seconds.

$s = \frac{1}{2}(a+b)h$

$4000 = \frac{1}{2}(120+T) \times 25$

$120 + T = \frac{4000 \times 2}{25} = 320 \Rightarrow T = 200$

The total time the car takes to travel from S to F is 200 s.

c The distance the car travels while accelerating is given by

$s = \frac{1}{2} \times 20 \times 25 = 250\,(\text{m})$

The car travels a further $(1500 - 250)\,\text{m} = 1250\,\text{m}$ at a constant speed. The time it takes to do this is given by

$1250 = 25t \Rightarrow t = 50$.

The car takes 70 s to reach P.

Hence the motorcycle takes 60 s to reach P.

In part **c**, you first have to find the time the car takes to get to P. There are two stages to this – the time for which the car accelerates (20 s, given) and the time for which it travels at a constant speed, which needs to be calculated.
The motorcycle then takes 10 s less than the sum of these two times.

d For the motorcycle

$u = 0, s = 1500, t = 60, v = ?$

$s = \left(\frac{u+v}{2}\right)t$

$1500 = \left(\frac{0+v}{2}\right)60 = 30v \Rightarrow v = \frac{1500}{30} = 50$

The speed of the motorcycle at P is 50 m s^{-1}.

This solution to part **d** makes no reference to a speed-time graph. It is not uncommon for some parts of a question to be better done using the properties of graphs and other parts to be better done using the one or more of the 5 kinematics formulae.

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Review Exercise
Exercise A, Question 24

Question:

Two cars *A* and *B* are travelling in the same direction along a motorway. They pass a warning sign at the same instant and, subsequently, arrive at a toll booth at the same instant.

Car *A* passes the warning sign at speed 24 m s^{-1}, continues at this speed for one minute, then decelerates uniformly, coming to rest at the toll booth.

Car *B* passes the warning sign at speed 30 m s^{-1}, continues at this speed for *T* seconds, then decelerates uniformly, coming to rest at the toll booth.

a On the same diagram, draw a speed–time graph to illustrate the motion of each car.

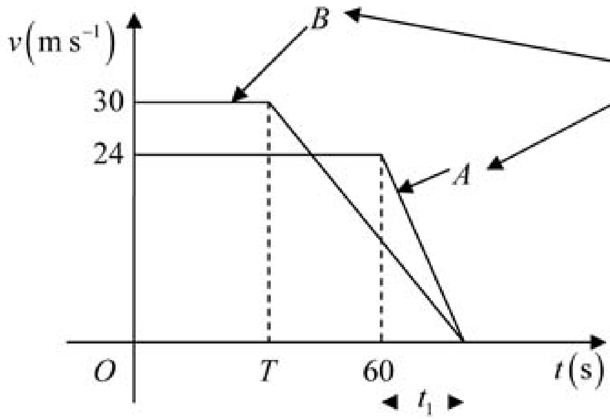
The distance between the warning sign and the toll booth is 1.56 km.

b Find the length of time for which *A* is decelerating.

c Find the value of *T*.

Solution:

a



You should label the graphs so that it is clear which represents car A and which represents car B.

b Let the time for which A is decelerating be t_1 s.

For car A

$$s = \frac{1}{2}(a+b)h$$

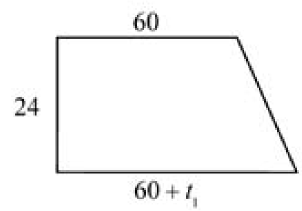
$$1560 = \frac{1}{2}(60+60+t_1)\times 24$$

$$= 12(120+t_1) = 1440 + 12t_1$$

$$t_1 = \frac{1560-1440}{12} = 10$$

The time for which A is decelerating is 10s.

The trapezium representing the distance moved by car A is



Putting the area of this trapezium equal to the distance travelled in metres, 1560, gives an equation which you can solve for t_1.

c Both cars take 70 s to move from the warning sign to the toll booth.

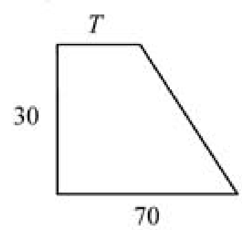
For Car B

$$s = \frac{1}{2}(a+b)h$$

$$1560 = \frac{1}{2}(T+70)\times 30 = 15T + 1050$$

$$T = \frac{1560-1050}{15} = 34$$

The trapezium representing the distance moved by car B is



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Review Exercise
Exercise A, Question 25

Question:

A train is travelling at 10 m s $^{-1}$ on a straight horizontal track. The driver sees a red signal 135 m ahead and immediately applies the brakes. The train immediately decelerates with constant deceleration for 12s, reducing its speed to 3 m s $^{-1}$. The driver then releases the brakes and allows the train to travel at a constant speed of 3 m s $^{-1}$ for a further 15 s. He then applies the brakes again and the train slows down with constant deceleration, coming to rest as it reaches the signal.

a Sketch a speed–time graph illustrating the motion of the train.

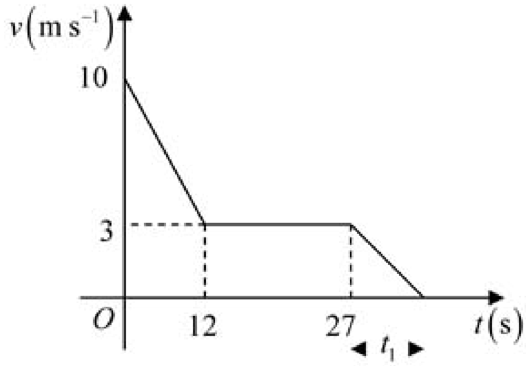
b Find the distance travelled by the train from the moment when the brakes are first applied to the moment when its speed first reaches 3 m s $^{-1}$.

c Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

d Sketch an acceleration–time graph illustrating the motion of the train.

Solution:

a

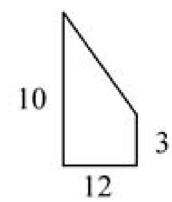


b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(3+10)\times 12 = 78$

The required distance is 78 m.

The trapezium representing the distance moved by the train during the initial deceleration is



c The distance travelled at constant speed 3 m s^{-1} is given by

$s = 3\times 15 = 45$

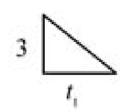
The distance travelled under the final deceleration is $(135-78-45)\text{m} = 12\text{ m}$.

Let the time taken for the final deceleration be t_1 s.

$12 = \frac{1}{2}t_1\times 3 \Rightarrow t_1 = 8$

The total time is $(12+15+8)\text{s} = 35\text{ s}$.

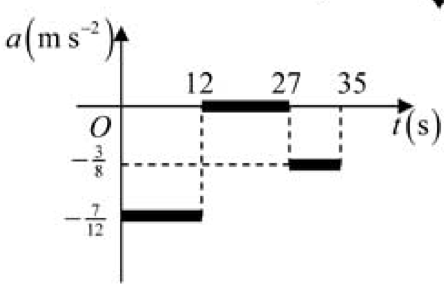
The distance travelled during the final deceleration is represented by the triangle



This distance is 12 m and use of the usual formula for the area of a triangle enables you to find t_1.

d The initial deceleration is $\frac{7}{12}\text{ m s}^{-2}$.

The final deceleration is $\frac{3}{8}\text{ m s}^{-2}$.



The decelerations can be written down using the key point that the gradient of a speed-time graph represents the acceleration.

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Review Exercise
Exercise A, Question 26

Question:

A straight stretch of railway line passes over a viaduct which is 600 m long. An express train on this stretch of line normally travels at a speed of 50 m s^{-1}. Some structural weakness in the viaduct is detected and engineers specify that all trains passing over the viaduct must do so at a speed of no more than 10 m s^{-1}. Approaching the viaduct, the train therefore reduces its speed from 50 m s^{-1} with constant deceleration 0.5 m s^{-2}, reaching a speed of precisely 10 m s^{-1} just as it reaches the viaduct. It then passes over the viaduct at a constant speed of 10 m s^{-1}. As soon as it reaches the other side, it accelerates to its normal speed of 50 m s^{-1} with constant acceleration 0.5 m s^{-2}.

a Sketch a speed–time graph to show the motion of the train during the period from the time when it starts to reduce speed to the time when it is running at full speed again.

b Find the total distance travelled by the train while its speed is less than 50 m s^{-1}.

c Find the extra time taken by the train for the journey due to the speed restriction on the viaduct.

Solution:

a To find time taken to decelerate

$u = 50,\ v = 10,\ a = -0.5,\ t = ?$

$v = u + at$

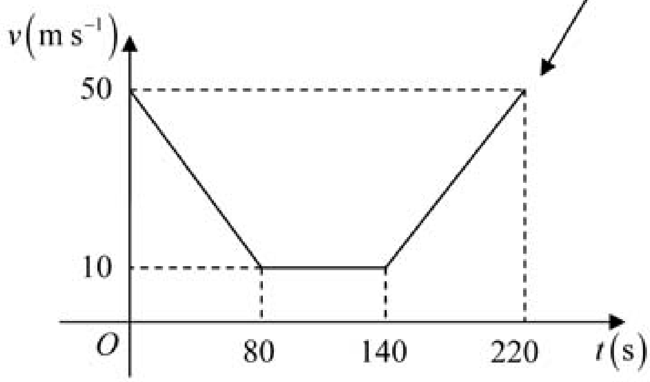
$10 = 50 - 0.5t \Rightarrow t = \dfrac{50-10}{0.5} = 80$

At the constant speed of $10\ \text{m s}^{-1}$

$s = ut$

$600 = 10 \times t \Rightarrow t = 60$

The acceleration is symmetric with the deceleration and takes 80 s.

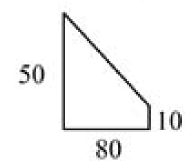


b $s = \frac{1}{2}(10+50)\times 80 + 60\times 10 + \frac{1}{2}(10+50)\times 80$

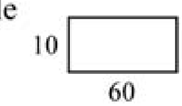
$= 2400 + 600 + 2400 = 5400$

The distance travelled by the train is 5400 m.

The total distance travelled is represented by two trapezia with dimensions



plus the rectangle



c At a constant speed of $50\ \text{m s}^{-1}$

distance = speed × time

$5400 = 50t \Rightarrow t = \dfrac{5400}{50} = 108$

The extra time is $(220-108)\text{s} = 112\ \text{s}$.

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Review Exercise
Exercise A, Question 27

Question:

A bus and a cyclist are moving along a straight horizontal road in the same direction. The bus starts at a bus stop *O* and moves with constant acceleration of 2 m s^{-2} until it reaches a maximum speed of 12 m s^{-1}. It then maintains this constant speed. The cyclist travels with a constant speed of 8 m s^{-1}. The cyclist passes *O* just as the bus starts to move. The bus later overtakes the cyclist at the point *A*.

a Show that the bus does not overtake the cyclist before it reaches its maximum speed.

b Sketch, on the same diagram, speed–time graphs to represent the motion of the bus and the cyclist as they move from *O* to *A*.

c Find the time taken for the bus and the cyclist to move from *O* to *A*.

d Find the distance *OA*.

Solution:

a Find the time for the bus to reach its maximum speed.

$u = 0, v = 12, a = 2, t = ?$

$v = u + at$

$12 = 0 + 2t \Rightarrow t = 6$

Find the distance travelled by the bus in reaching its maximum speed.

$u = 0, v = 12, a = 2, s = ?$

$v^2 = u^2 + 2as$

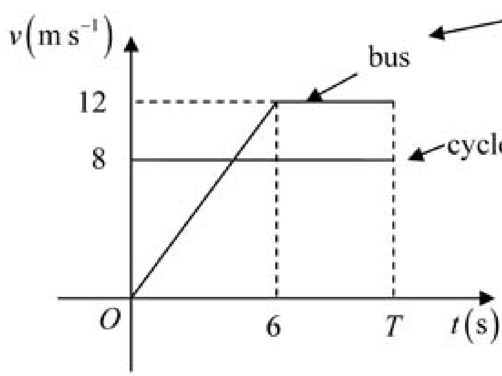
$12^2 = 0^2 + 4s \Rightarrow s = 36 \ (\text{m})$

In 6 s, the distance travelled by the cyclist is given by distance = speed × time.

$= 8 \times 6 = 48 \, (\text{m})$

As 36 m is less than 48 m the bus has not overtaken the cyclist.

b



You should label the graphs so that it is clear which represents the bus and which represents the cycle.

c Let T seconds be the time for the bus and the cyclist to move from O to A.

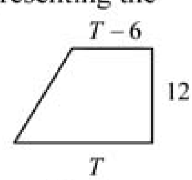
$\frac{1}{2}(T - 6 + T) \times 12 = 8T$

$6(2T - 6) = 12T - 36 = 8T$

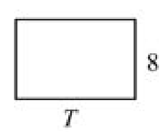
$4T = 36 \Rightarrow T = 9$

The time taken is 9 s.

The area of the trapezium representing the distance travelled by the bus



must equal the rectangle representing the distance travelled by the cycle



d For the cycle, distance = speed × time

$= 8 \times 9 \text{ m} = 72 \text{ m}$

$OA = 72 \text{ m}$

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Review Exercise
Exercise A, Question 28

Question:

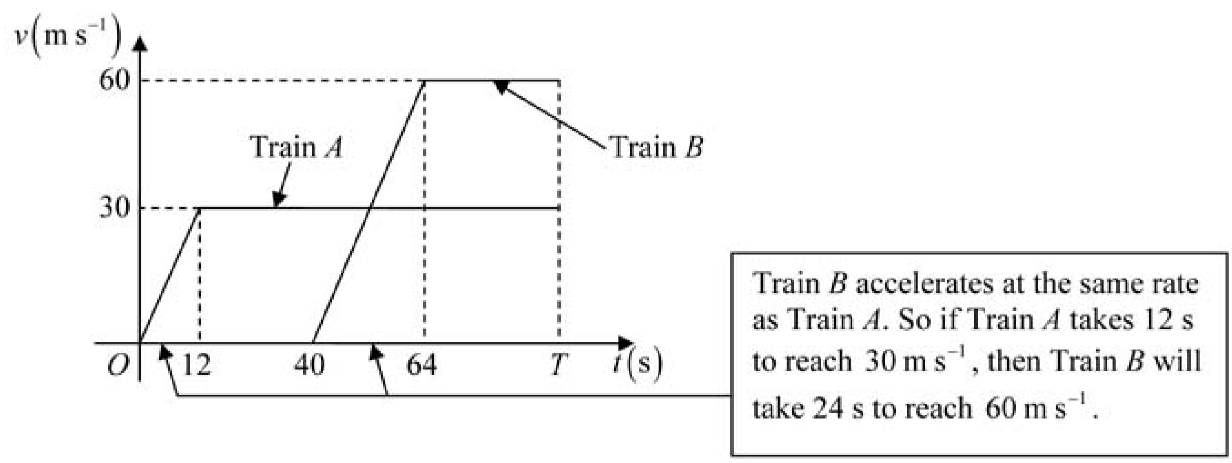
Two trains A and B run on parallel straight tracks. Initially both are at rest in a station and level with each other. At time t = 0, A starts to move. It moves with constant acceleration for 12 s up to a speed of 30 m s^{-1}, and then moves at a constant speed of 30 m s^{-1}. Train B starts to move in the same direction as A when t = 40, where t is measured in seconds. It accelerates with the same initial acceleration as A, up to a speed of 60 m s^{-1}. It then moves at a constant speed of 60 m s^{-1}. Train B overtakes A after both trains have reached their maximum speed. Train B overtakes A when t = T.

a Sketch, on the same diagram, the speed–time graphs of both trains for $0 \le t \le T$.

b Find the value of T.

Solution:

a



b The distance travelled by Train A in T seconds is given by

$$s = \frac{1}{2}(T - 12 + T) \times 30 = 15(2T - 12)$$

The distance travelled by Train B in T seconds is given by

$$s = \frac{1}{2}(T - 64 + T - 40) \times 60 = 20(2T - 104)$$

At the point of overtaking the distances are equal.

You use the fact that the two trapezia have the same area to form an equation in T, which you solve.

$$15(2T - 12) = 30(2T - 104)$$
$$30T - 180 = 60T - 3120$$
$$30T = 2940$$
$$T = \frac{2940}{30} = 98$$

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 29

Question:

A train starts from rest at a station, accelerates uniformly to its maximum speed of 15 m s^{-1}, travels at this speed for a time, and then decelerates uniformly to rest at the next station. The distance from station to station is 1260 m, and the time spent travelling at the maximum speed is three-quarters of the total journey time.

a Sketch a speed–time graph to illustrate this information.

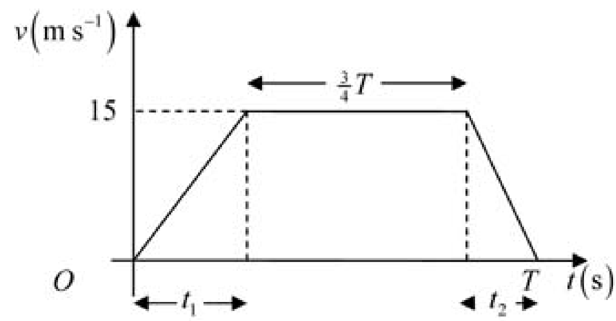
b Find the total journey time.

Given also that the magnitude of the deceleration is twice the magnitude of the acceleration,

c find the magnitude of the acceleration.

Solution:

a



b Let the total time for the journey be T seconds.

$$s = \frac{1}{2}(a+b)h$$

$$1260 = \frac{1}{2}\left(\frac{3}{4}T + T\right) \times 15$$

$$\frac{7}{4}T = \frac{2 \times 1260}{15} = 168$$

$$T = \frac{4 \times 168}{7} = 96$$

The total time for the journey is 96 s.

c Let the time taken accelerating be t_1 seconds.

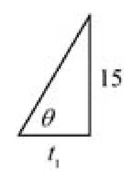
Let the time taken decelerating be t_2 seconds.

$$t_1 + t_2 = \frac{1}{4}T = 24 \quad \ldots \ldots (1)$$

The acceleration is $\frac{15}{t_1}$ s

The acceleration is represented by the gradient of the line .

Gradient $= \frac{15}{t_1}$



The deceleration is $\frac{15}{t_2}$ s

You get an expression for the deceleration in a similar way to that used for the acceleration.

The magnitude of the deceleration is twice the magnitude of the acceleration.

$$\frac{15}{t_2} = 2 \times \frac{15}{t_1} \Rightarrow t_2 = \frac{1}{2}t_1 \quad \ldots \ldots (2)$$

Substitute (2) into (1)

$$t_1 + \frac{1}{2}t_1 = \frac{3}{2}t_1 = 24 \Rightarrow t_1 = \frac{2}{3} \times 24 = 16$$

The acceleration is $\frac{15}{t_1} = \frac{15}{16}\ \mathrm{m\,s^{-2}}$.

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Review Exercise
Exercise A, Question 30

Question:

The brakes of a train, which is travelling at 108 km h^{-1}, are applied as the train passes a point *A*. The brakes produce a retardation of magnitude $3f$ m s^{-2} until the speed of the train is reduced to 36 km h^{-1}. The train travels at this speed for a distance and is then uniformly accelerated at f m s^{-2} until it again reaches the speed 108 km h^{-1} as it passes point *B*. The time taken by the train in travelling from *A* to *B*, a distance of 4 km, is 4 minutes.

a Sketch a speed–time graph to illustrate the motion of the train from *A* to *B*.

b Find the value of *f*.

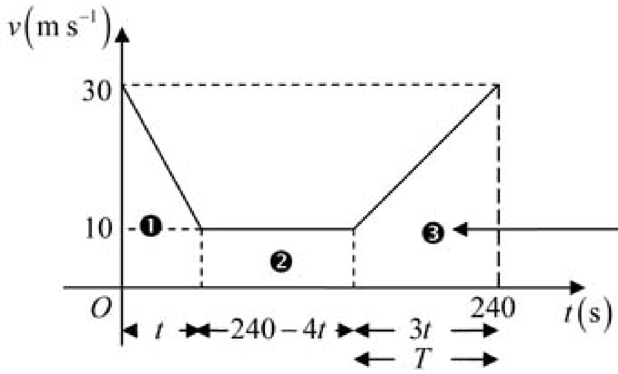
c Find the distance travelled at 36 km h^{-1}.

Solution:

a $108 \text{ km h}^{-1} = \dfrac{108 \times 1000}{3600} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$

You must work in a consistent set of units. The speeds are in km and hours, the accelerations in m and s. Reduce everything to m and s.



b Let t seconds be the time the train takes to retard.

Then $3f = \dfrac{20}{t} \Rightarrow f = \dfrac{20}{3t}$ …… (1)

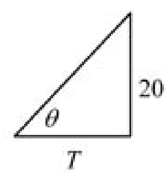
Let T seconds be the time takes to accelerate.

Then $f = \dfrac{20}{T}$ …… (2)

Accelerations and deceleration are the gradients of line. Here, while accelerating ,

$f =$ gradient

$= \dfrac{20}{T}$



From (1) and (2) $T = 3t$ and the time travelled at constant speed is $240 - 4t$ seconds.

$$\frac{1}{2}(10+30)t + 10(240-4t) + \frac{1}{2}(10+30)3t = 4000$$

The areas marked ❶, ❷ and ❸ in the speed-time diagram added together represent the distance travelled. ❶ and ❸ are trapezia; ❷ is a rectangle.

$$20t + 2400 - 40t + 60t = 4000$$
$$40t = 1600 \Rightarrow t = 40$$

From (2) above

$$f = \frac{20}{T} = \frac{20}{3t} = \frac{20}{120} = \frac{1}{6}$$

c At constant speed, distance = speed × time

$$s = 10 \times (240 - 4t) = 10 \times (240 - 4 \times 40)$$
$$= 10 \times 80 = 800$$

The distance travelled at 36 km h^{-1} is 800 m.

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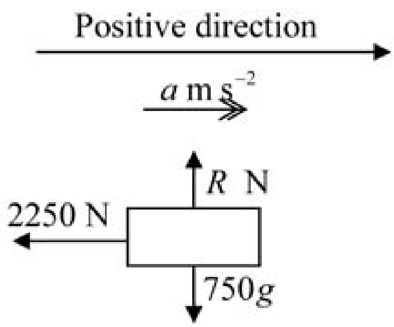
Review Exercise
Exercise A, Question 31

Question:

A car of mass 750 kg, moving along a level straight road, has its speed reduced from 25 m s^{-1} to 15 m s^{-1} by brakes which produce a constant retarding force of 2250 N. Calculate the distance travelled by the car as its speed is reduced from 25 m s^{-1} to 15 m s^{-1}.

Solution:



$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \quad -2250 = 750a$$

The retarding force is slowing the car down and is in the negative direction. So, in the positive direction, the force is -2250.

$$a = -\frac{2250}{750} = -3$$

$$u = 25,\ v = 15,\ a = -3,\ s = ?$$

$$v^2 = u^2 + 2as$$

There is no t here, so you choose the formula without t, $v^2 = u^2 + 2as$.

$$15^2 = 25^2 - 6s$$

$$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\frac{2}{3}$$

The distance travelled by the car as its speed is reduced is $66\frac{2}{3}$ m s^{-1}.

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Review Exercise
Exercise A, Question 32

Question:

A particle P is moving with constant acceleration along a straight horizontal line ABC, where $AC = 24$ m. Initially P is at A and is moving with speed 5 m s^{-1} in the direction AB. After 1.5 s, the direction of motion of P is unchanged and P is at B with speed 9.5 m s^{-1}.

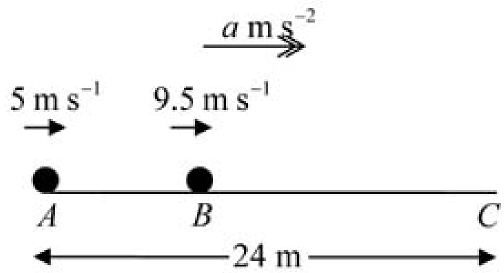
a Show that the speed of P at C is 13 m s^{-1}.

The mass of P is 2 kg. When P reaches C, an impulse of magnitude 30 Ns is applied to P in the direction CB.

b Find the velocity of P immediately after the impulse has been applied, stating clearly the direction of motion of P at this instant.

Solution:

a



From A to B

$u = 5, v = 9.5, t = 1.5, a = ?$

$v = u + at$

$9.5 = 5 + 1.5a$

$a = \frac{9.5 - 5}{1.5} = 3$

From A to C

$u = 5, s = 24, a = 3, v = ?$

$v^2 = u^2 + 2as$

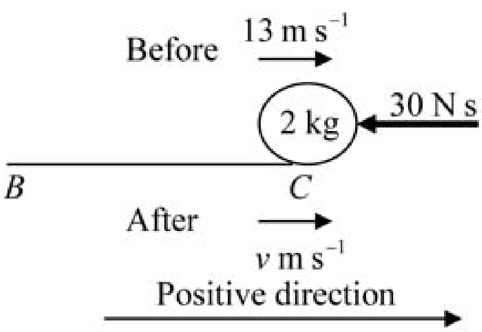
$= 5^2 + 2 \times 3 \times 24 = 169$

$v = \sqrt{169} = 13$

The speed of P at C is 13 m s^{-1}, as required.

You need to find the acceleration, a, first by considering the motion from A to B. Then you can proceed to find the speed at C by considering the motion from A to C.

b



$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$-30 = 2 \times v - 2 \times 13$

$2v = -30 + 26 = -4$

$v = -2$

The velocity of P immediately after the impulse has been applied is 2 m s^{-1} in the direction $\overrightarrow{CB}$.

Impulse is a vector quantity and you must always consider its direction. If the direction from B to C is taken as the positive direction then an impulse of 30 N s in the direction from C to B must be taken as -30.

The answer $v = -2$ shows that the velocity is in the negative direction, that is from C to B.

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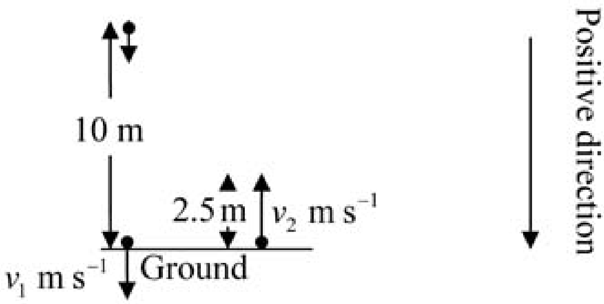
Review Exercise
Exercise A, Question 33

Question:

A ball of mass 0.3 kg is released at rest from a point at a height of 10 m above horizontal ground. After hitting the ground the ball rebounds to a height of 2.5 m.

Calculate the magnitude of the impulse exerted by the ground on the ball.

Solution:



As ball descends

$u = 0,\ a = 9.8,\ s = 10,\ v = v_1$

$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2 \times 10 \times 9.8 = 196$

$v_1 = \sqrt{196} = 14$

The ball is released from rest 10 m above the ground. The first step is to calculate the speed with which the ball strikes the ground.

After rebound

$v = 0,\ a = 9.8,\ s = -2.5,\ u = v_2$

$v^2 = u^2 + 2as$

$0^2 = v_2^2 + 2 \times 9.8 \times (-2.5) \Rightarrow v_2^2 = 49$

$v_2 = -\sqrt{49} = -7$

You must then use the fact that the ball reaches a maximum height of 2.5 m to find the velocity with which it rebounds from the ground.

As it rebounds from the ground, the ball is moving upwards. That is in the negative direction. You must take the negative square root of 49, which is -7.

$I = mv_2 - mv_1$

$\quad = 0.3 \times (-7) - 0.3 \times 14 = -6.9$

The magnitude of the impulse is 6.9 N.

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Review Exercise
Exercise A, Question 34

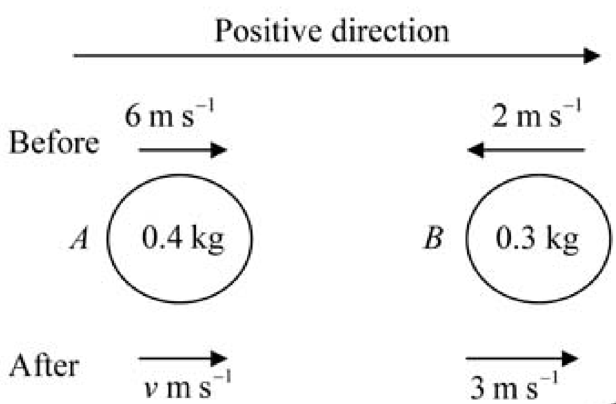
Question:

Two particles *A* and *B* have mass 0.4 kg and 0.3 kg respectively. They are moving in opposite directions on a smooth horizontal table and collide directly. Immediately before the collision, the speed of *A* is 6 m s $^{-1}$ and the speed of *B* is 2 m s $^{-1}$. As a result of the collision, the direction of motion of *B* is reversed and its speed immediately after the collision is 3 m s $^{-1}$. Find

a the speed of *A* immediately after the collision, stating clearly whether the direction of motion of *A* is changed by the collision,

b the magnitude of the impulse exerted on *B* in the collision, stating clearly the units in which your answer is given.

Solution:



The total linear momentum before impact must equal the total linear momentum after impact.

a Conservation of linear momentum

The velocity of *B* before impact is in the negative direction so it must be entered as -2 in any equations involving linear momentum.

$$0.4\times 6+0.3\times(-2)=0.4\times v+0.3\times 3$$
$$2.4-0.6=0.4v+0.9$$
$$0.4v=2.4-0.6-0.9=0.9$$
$$v=\frac{0.9}{0.4}=2.25$$

The speed of *A* after the collision is 2.25 m s^{-1}.
The direction of motion of *A* is unchanged.

The velocity of *A* is positive (2.25 m s^{-1}) after impact and it was positive (6 m s^{-1}) before impact. So the direction of motion of *A* is unchanged.

b For B, $\mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=0.3\times 3-0.3\times(-2)$$
$$=0.9+0.6=1.5$$

The magnitude of the impulse exerted on *B* is 1.5 N s.

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Review Exercise
Exercise A, Question 35

Question:

A railway truck S of mass 2000 kg is travelling due east along a straight horizontal track with constant speed 12 m s $^{-1}$. The truck S collides with a truck T which is travelling due west along the same track as S with constant speed 6 m s $^{-1}$. The magnitude of the impulse of T on S is 28 800 Ns.

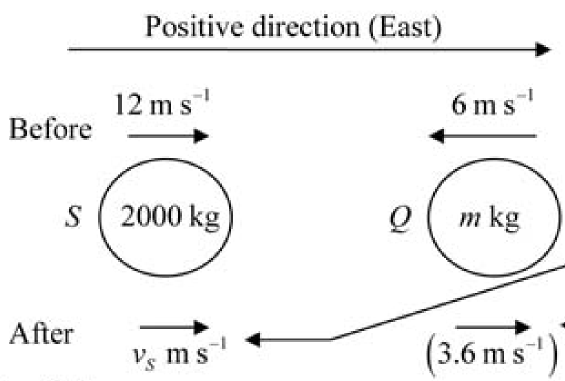
a Calculate the speed of S immediately after the collision.

b State the direction of motion of S immediately after the collision.

Given that, immediately after the collision, the speed of T is 3.6 m s $^{-1}$, and that T and S are moving in opposite directions,

c calculate the mass of T.

Solution:



You do not know which direction S will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction After you have worked out v, the sign of v will tell you the direction S is moving in.

You may not be sure, when starting the question, which direction I is moving in after the impact. You can leave it blank at this stage a fill it in after answering part **b**.

a For S, $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$-28\,800 = 2000 \times v_S - 2000 \times 12$$

$$2000 v_S = -28\,800 + 24\,000 = -4800$$

$$v_S = -\frac{4800}{2000} = -2.4$$

The speed of S immediately after the collision is 2.4 m s^{-1}.

b Immediately after the collision S is moving due west.

The sign of v is negative, so S is moving in the negative direction. In this solution, the positive direction has been taken as east, so S is now moving west.

c Conservation of linear momentum

$$2000 \times 12 + m \times (-6) = 2000 \times (-2.4) + m \times 3.6$$

$$9.6m = 24\,000 + 4800 = 28\,800 \Rightarrow m = \frac{28\,800}{9.6} = 3000$$

The mass of T is 3000 kg.

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Review Exercise
Exercise A, Question 36

Question:

Two particles *A* and *B*, of mass 0.5 kg and 0.4 kg respectively, are travelling in the same straight line on a smooth horizontal table. Particle *A*, moving with speed 3 m s^{-1}, strikes particle *B*, which is moving with speed 2 m s^{-1} in the same direction. After the collision *A* and *B* are moving in the same direction and the speed of *B* is 0.8 m s^{-1} greater than the speed of *A*.

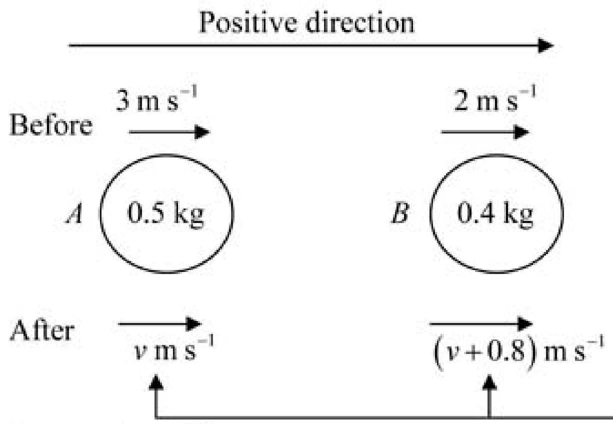
a Find the speed of *A* and the speed of *B* after the collision.

b Show that *A* loses momentum 0.4 N s in the collision.

Particle *B* later hits an obstacle on the table and rebounds in the opposite direction with speed 1 m s^{-1}.

c Find the magnitude of the impulse received by *B* in this second impact.

Solution:



You need to translate the statement that 'the speed of B is 0.8 m s^{-1} greater than the speed of A' into algebra. If the speed of A after the collision is $v \text{ m s}^{-1}$ then the speed of B is 0.8 m s^{-1} greater; that is $(v+0.8)\text{m s}^{-1}$.

a Conservation of linear momentum

$$0.5\times 3+0.4\times 2=0.5\times v+0.4(v+0.8)$$
$$1.5+0.8=0.5v+0.4v+0.32$$
$$0.9v=1.5+0.8-0.32=1.98$$
$$v=\frac{1.98}{0.9}=2.2$$

All velocities in this part are in the positive direction.

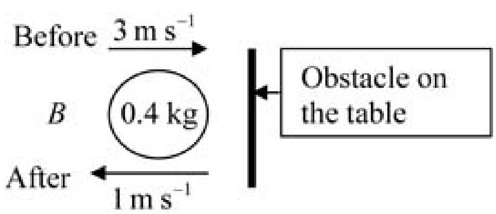
The speed of A after the collision is 2.2 m s^{-1}.
The speed of B after the collision is $(2.2+0.8)\text{ m s}^{-1}=3\text{ m s}^{-1}$.

To find the speed of B add 0.8 m s^{-1} to the speed of A.

b The momentum of A before the collision is given by
$$mu=0.5\times 3 \text{ N s}=1.5 \text{ N s}$$
The momentum of A after the collision is given by
$$mv=0.5\times 2.2 \text{ N s}=1.1 \text{ N s}$$
A loses momentum $(1.5-1.1)\text{N s}=0.4 \text{ N s}$, as required.

The momentum of a particle is its mass times its velocity. Momentum is a vector quantity.

c



For B, before and after the second impact
$$\mathbf{I}=m\mathbf{v}-m\mathbf{u}$$
$$=0.4\times(-1)-0.4\times 3$$
$$=-1.6$$
The magnitude of the impulse received by B in this second impact is 1.6 N s.

Left to right has been taken as the positive direction throughout the question. The impulse on B is negative as, as the situation is drawn here, the impulse on B is in the direction from right to left.

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Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 37

Question:

Two particles *A* and *B*, of mass 3 kg and 2 kg respectively, are moving in the same direction on a smooth horizontal table when they collide directly. Immediately before the collision, the speed of *A* is 4 m s^{-1} and the speed of *B* is 1.5 m s^{-1}. In the collision, the particles join to form a single particle *C*.

a Find the speed of *C* immediately after the collision.

Two particles *P* and *Q* have mass 3 kg and *m* kg respectively. They are moving towards each other in opposite directions on a smooth horizontal table. Each particle has speed 4 m s^{-1}, when they collide directly. In this collision, the direction of motion of each particle is reversed. The speed of *P* immediately after the collision is 2 m s^{-1} and the speed of *Q* is 1 m s^{-1}

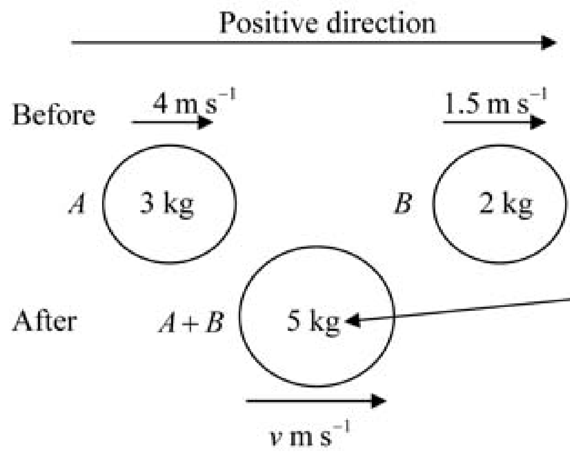
b Find

i the value of *m*,

ii the magnitude of the impulse exerted on *Q* in the collision.

Solution:

a



After the collision A (of mass 3 kg) and B (of mass 2 kg) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, 5 kg.

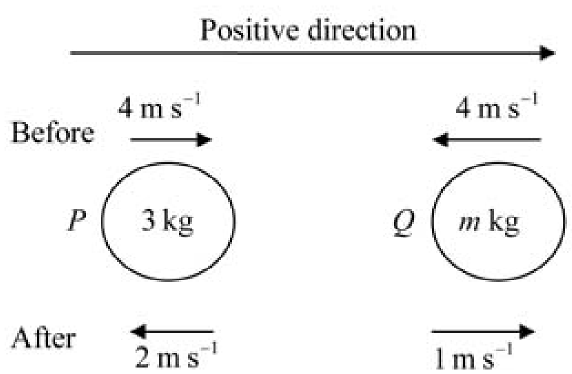
Conservation of linear momentum

$4 \times 3 + 2 \times 1.5 = 5 \times v$

$$12 + 3 = 5v \Rightarrow v = \frac{15}{5} = 3$$

The speed of C immediately after the collision is 3 m s^{-1}.

b



(i) Conservation of linear momentum

$$3 \times 4 + m \times (-4) = 3 \times (-2) + m \times 1$$

In the equation for the conservation of momentum, you must give the velocities in the negative direction a negative sign,

$$12 - 4m = -6 + m \Rightarrow 5m = 18$$

$$m = \frac{18}{5} = 3.6$$

(ii) For Q, $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$$I = 3.6 \times 1 - 3.6 \times (-4)$$

$$= 3.6 + 14.4 = 18$$

The magnitude of the impulse exerted on Q in the collision is 18 N s.

As the magnitude of the impulse exerted on P is the same as the magnitude of the impulse exerted on Q, you could equally correctly work out the change in linear momentum of P. The working then would be $I = 3 \times (-2) - 3 \times 4 = -18$, which gives the same magnitude, 18 N s..

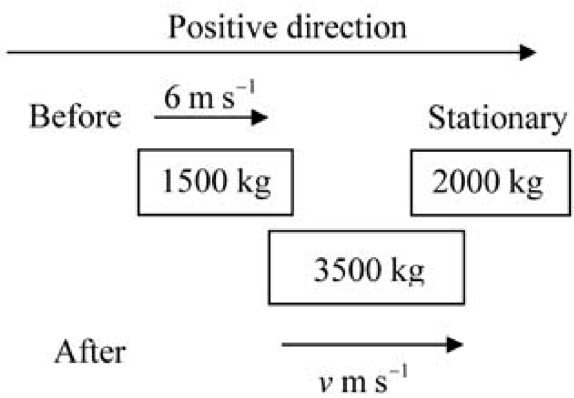
Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 38

Question:

A railway truck, of mass 1500 kg and travelling with a speed 6 m s^{-1} along a horizontal track, collides with a stationary truck of mass 2000 kg. After the collision the two trucks move on together, coming to rest after 12 seconds. Calculate the magnitude of the constant force resisting their motion after the collision.

Solution:



Conservation of linear momentum

$1500 \times 6 = 3500v$

$v = \frac{1500 \times 6}{3500} = \frac{18}{7}$

To find deceleration

$u = \frac{18}{7}, v = 0, t = 12, a = ?$

$v = u + at$

$0 = \frac{18}{7} + 12a \Rightarrow a = -\frac{18}{7} \times \frac{1}{12} = -\frac{3}{14}$

To find the magnitude of the resisting force, F, say

$\mathbf{F} = m\mathbf{a}$

$F = 3500 \times -\frac{3}{14} = -750$

The magnitude of the resisting force is 750 N.

You need to have a plan when tackling this sort of question. The question has not broken the problem down into separate parts and you need to make your own plan. You have to find the three steps for yourself.
1. Use conservation of linear momentum to find the velocity of the trucks after the collision.
2. Use a kinematics formula to find the deceleration.
3. Use Newton's Second law with your result from step 2 to find the force.

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Review Exercise
Exercise A, Question 39

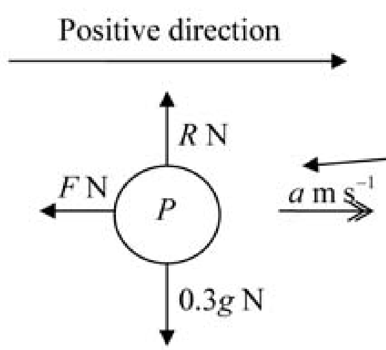
Question:

A particle P of mass 0.3 kg is moving in a straight line on a rough horizontal plane. The speed of P decreases from 7.5 m s $^{-1}$ to 4 m s $^{-1}$ in time T seconds. Given the coefficient of friction between P and the plane is $\frac{1}{7}$, find

a the magnitude of the frictional force opposing the motion of P,

b the value of T.

Solution:



You should begin by drawing a diagram which shows all of the forces acting on P and its acceleration.

a $R(\uparrow)\quad R - 0.3g = 0 \Rightarrow R = 0.3g$

$$F = \mu R$$
$$= \frac{1}{7} \times 0.3 \times 9.8 = 0.42$$

The magnitude of the frictional force opposing the motion of P is 0.42 N.

b $\mathbf{F} = m\mathbf{a}$

$R(\rightarrow)\ -F = 0.3a$

Using the result of part **a**

$-0.42 = 3a \Rightarrow a = -\frac{0.42}{0.3} = -1.4$

$u = 7.5, v = 4, a = -1.4, T = ?$

$v = u + at$

$4 = 7.5 - 1.4T$

$T = \frac{7.5 - 4}{1.4} = 2.5$

You are asked to find T but before you can use $v = u + at$, you have to find the value of a, using Newton's second law. As the particle is slowing down, a is negative.

Solutionbank M1

Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 40

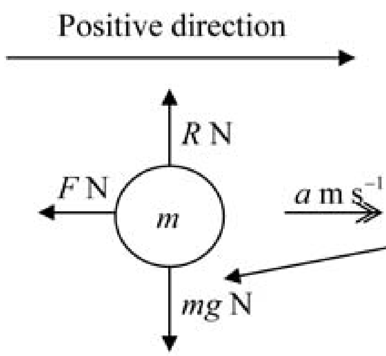
Question:

A small stone moves horizontally in a straight line across the surface of an ice rink. The stone is given an initial speed of 7 m s $^{-1}$. It comes to rest after moving a distance of 10 m. Find

a the deceleration of the stone while it is moving,

b the coefficient of friction between the stone and the ice.

Solution:



You are given no value for mass of the small stone and you will need to have an expression for the weight of the stone. Let the mass of the stone be m kg, then the weight of the stone is mg N.

a

$u = 7, v = 0, s = 10, a = ?$

$v^2 = u^2 + 2as$

$0^2 = 7^2 + 2 \times a \times 10$

$a = -\frac{49}{20} = -2.45$

The deceleration of the stone is 2.45 m s^{-1}.

b

$R(\uparrow)$ $R - mg = 0 \Rightarrow R = mg$

$F = \mu R = \mu mg$

$\mathbf{F} = m\mathbf{a}$

$R(\rightarrow)$ $-F = ma$

$-\mu mg = m \times (-2.45)$

$\mu = \frac{2.45\cancel{m}}{9.8\cancel{m}} = 0.25$

The m 'cancels' at the end of the question. This result would be the same with a small stone of any mass.

The coefficient of friction between the stone and the ice is 0.25.

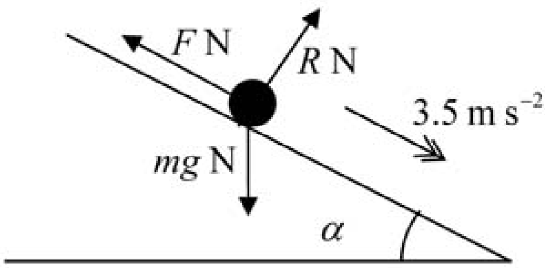
Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 41

Question:

A rough plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. A particle slides with acceleration 3.5 m s $^{-1}$ down a line of greatest slope of this plane. Calculate the coefficient of friction between the particle and the plane.

Solution:



$R(\nearrow)$ $\quad R - mg\cos\alpha = 0$

$$R = mg\cos\alpha = \tfrac{4}{5}mg$$

Friction is limiting

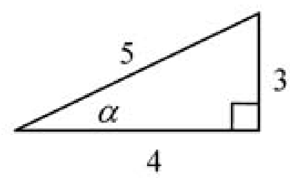
$$F = \mu R = \tfrac{4}{5}\mu mg$$

$R(\searrow)$ $\quad mg\sin\alpha - F = ma$

$$\tfrac{3}{5}\cancel{m}g - \tfrac{4}{5}\mu \cancel{m}g = \cancel{m}3.5$$

$$\mu = \frac{\tfrac{3}{5}\times 9.8 - 3.5}{\tfrac{4}{5}\times 9.8} = 0.3035\ldots$$

The coefficient of friction between the particle and the plane is 0.30 (2 s.f.).



You can see from the above diagram that if $\tan\alpha = \frac{3}{4}$, then $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

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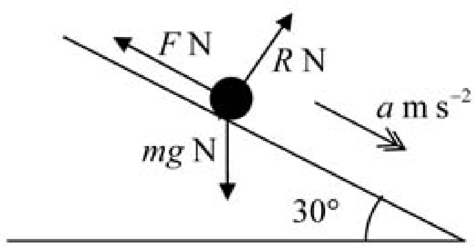
Review Exercise
Exercise A, Question 42

Question:

A particle moves down a line of greatest slope of a rough plane which is inclined at 30 ° to the horizontal. The particle starts from rest and moves 3.5 m in time 2 s. Find the coefficient of friction between the particle and the plane.

Solution:



$u = 0, s = 3.5, t = 2, a = ?$

$s = ut + \frac{1}{2}at^2$

$3.5 = 0 + \frac{1}{2}a \times 2^2 = 2a$

$a = \frac{3.5}{2} = 1.75$

$R(\nearrow) \quad R - mg\cos 30° = 0 \Rightarrow R = mg\cos 30°$

Friction is limiting

$F = \mu R = \mu mg \cos 30°$

$R(\searrow) \quad mg\sin 30° - F = ma$

$\cancel{m}g\sin 30° - \mu \cancel{m}g\cos 30° = \cancel{m} \times 1.75$

$\mu = \frac{9.8\sin 30° - 1.75}{9.8\cos 30°} = 0.3711 \ \ldots$

The coefficient of friction between the particle and the plane is 0.37 (2 s.f.).

As often happens, the m which you had to introduce at the beginning, "cancels" because it is a common factor of all of the terms in the equation.

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Review Exercise
Exercise A, Question 43

Question:

A stone *S* is sliding on ice. The stone is moving along a straight line *ABC*, where $AB = 24$ m and $AC = 30$ m. The stone is subject to a constant resistance to motion of magnitude 0.3 N. At *A* the speed of *S* is 20 m s^{-1}, and at *B* the speed of *S* is 16 m s^{-1}. Calculate

a the deceleration of *S*,

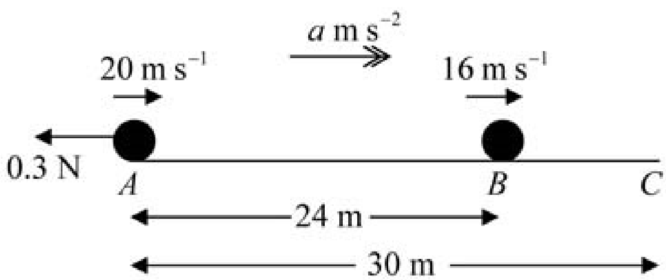
b the speed of *S* at *C*.

c Show that the mass of *S* is 0.1 kg.

At *C*, the stone *S* hits a vertical wall, rebounds from the wall and then slides back along the line *CA*. The magnitude of the impulse of the wall on *S* is 2.4 N s and the stone continues to move against a constant resistance of 0.3 N.

d Calculate the time between the instant that *S* rebounds from the wall and the instant that *S* comes to rest.

Solution:



a From *A* to *B*

$u = 20, v = 16, s = 24, a = ?$

$v^2 = u^2 + 2as$

$16^2 = 20^2 + 48s$

$s = \dfrac{16^2 - 20^2}{48} = -3$

The deceleration of *S* is 3 m s^{-2}.

b From *A* to *C*

$u = 20, s = 30, a = -3, v = ?$

$v^2 = u^2 + 2as$

$= 20^2 + 2 \times -3 \times 30 = 400 - 180 = 220$

$v = \sqrt{220} \approx 14.8$

The speed of *S* at *C* is 14.8 m s^{-1} (3 s.f.).

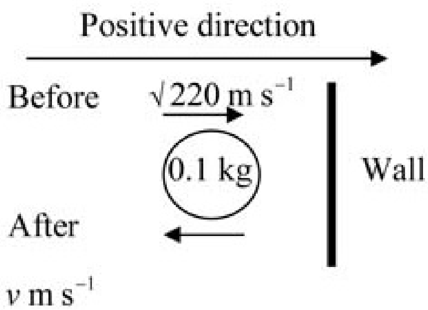
No accuracy is specified in this question and no numerical value of *g* is used. Where there is no exact answer, it is reasonable for you to give your answers to 3 significant figures.

c $\mathbf{F} = m\mathbf{a}$

$-0.3 = m \times -3 \Rightarrow m = \dfrac{0.3}{3} = 0.1$

The mass of *S* is 0.1 kg, as required.

d



$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$

$-2.4 = 0.1 \times (-v) - 0.1 \times \sqrt{220}$

$0.1v = 2.4 - 0.1 \times \sqrt{220} = 0.916\,76\ldots$

$v = 9.167\,6\ldots$

S now slows down with deceleration 3 m s^{-2}.

$u = 9.167\,6\ldots,\ v = 0,\ a = -3,\ t = ?$

$v = u + at \Rightarrow 0 = 9.167\,6\ldots - 3t$

$t = 3.055\ldots$

The time taken to slow to rest is 3.06 s (3 s.f).

It is quite common for the final velocity in one calculation, here the velocity after the impact with the wall, to become the initial velocity in the next part of the question, here the deceleration. So v in one calculation is u in the next.

The equation of motion has not changed since part (c), except that it has reversed direction. The resistance is still 0.3 N and the mass is, obviously, unchanged.

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Review Exercise
Exercise A, Question 44

Question:

A railway truck P of mass 1500 kg is moving on a straight horizontal track. The truck P collides with a truck Q of 2500 kg at a point A. Immediately before the collision, P and Q are moving in the same direction with speeds 10 m s $^{-1}$ and 5 m s $^{-1}$ respectively. Immediately after the collision, the direction of motion of P is unchanged and its speed is 4 m s $^{-1}$. By modelling the trucks as particles,

a show that the speed of Q immediately after the collision is 8.6 m s $^{-1}$.

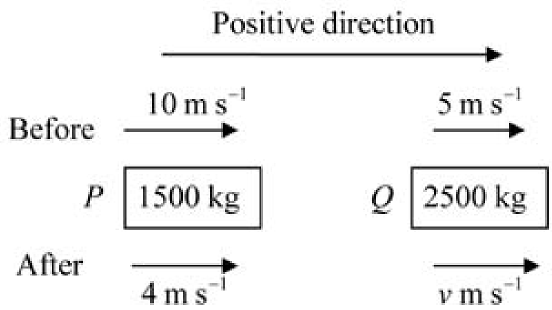
After the collision at A, the truck P is acted upon by a constant braking force of magnitude 500 N. The truck P comes to rest at the point B.

b Find the distance AB.

After the collision Q continues to move with constant speed 8.6 m s $^{-1}$.

c Find the distance between P and Q at the instant when P comes to rest.

Solution:



a Conservation of linear momentum

$1500\times10+2500\times5=1500\times4+2500v$

$15\,000+12\,500=6000+2500v$

$$v=\frac{15\,000+12\,500-6000}{2500}=\frac{21\,500}{2500}=8.6$$

The speed of Q immediately after the collision is 8.6 m s^{-1}, as required.

b For P

$\mathbf{F}=m\mathbf{a}$

$R(\rightarrow)\ -500=1500a \Rightarrow a=-\frac{1}{3}$

The only force acting on P in the horizontal direction is the braking force of 500 N.

$u=4, v=0, a=-\frac{1}{3}, s=?$

$v^2=u^2+2as$

$0^2=4^2-\frac{2}{3}s \Rightarrow s=\frac{3}{2}\times16=24$

The distance AB is 24 m.

c The time taken for P to come to rest is given by

$u=4, v=0, a=-\frac{1}{3}, t=?$

$v=u+at$

$0=4-\frac{1}{3}t \Rightarrow t=12$

The distance travelled by Q is given by

distance = speed × time

$s=8.6\times12=103.2$

Before you can find the distance travelled by truck Q as truck P comes to rest, you will have to find the time taken by P to come to rest. As Q is travelling at a constant speed, the distance it travels is found using distance = speed × time .

The distance between P and Q at the instant when P comes to rest is $(103.2-24)\text{ m}=79.2\text{ m}$.

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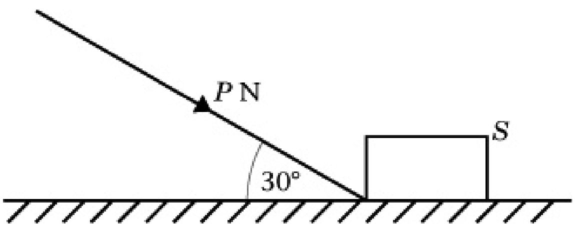
Review Exercise
Exercise A, Question 45

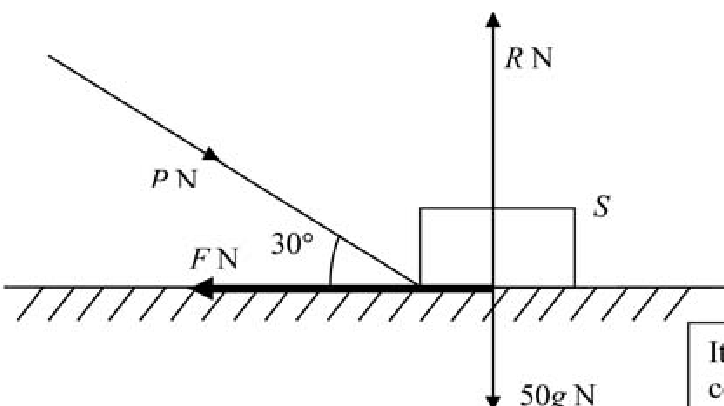
Question:



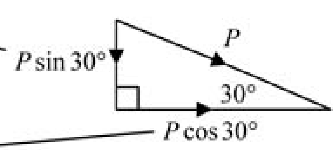
A heavy suitcase S of mass 50 kg is moving along a horizontal floor under the action of a force of magnitude P newtons. The force acts at 30 ° to the floor, as shown in the figure, and S moves in a straight line at constant speed. The suitcase is modelled as a particle and the floor as a rough horizontal plane. The coefficient of friction between S and the floor is $\frac{3}{5}$.

Calculate the value of P.

Solution:



It is a common error to omit the component of P when resolving vertically. A force at 30° to the horizontal has components in both the vertical and horizontal directions.



$R(\uparrow)$ $\quad R - 50g - P\sin 30° = 0$

$$R = 50g + P\sin 30°$$

Friction is limiting

$$F = \mu R = \tfrac{3}{5}(50g + P\sin 30°)$$

As S is moving horizontally at a constant speed, it has zero acceleration.

$R(\rightarrow)$ $\quad P\cos 30° - F = 50 \times 0$

$$P\cos 30° - 0.6(50g + P\sin 30°) = 0$$

$$P\cos 30° - 30g + 0.6P\sin 30° = 0$$

$$P(\cos 30° + 0.6\sin 30°) = 30g$$

Collect the terms in P on one side of the equation and the constant terms (there is only one here, $30g$) on the other.

$$P = \frac{30 \times 9.8}{\cos 30° + 0.6\sin 30°} = 252.1 \ \ldots$$

$$= 250 \ \text{(2 s.f.)}$$

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Review Exercise
Exercise A, Question 46

Question:

An engine of mass 25 tonnes pulls a truck of mass 10 tonnes along a railway line. The frictional resistances to the motion of the engine and the truck are modelled as constant and of magnitude 50 N per tonne. When the train is travelling horizontally the tractive force exerted by the engine is 26 kN. Modelling the engine and the truck as particles and the coupling between the engine and the truck as a light horizontal rod, calculate

a the acceleration of the engine and the truck,

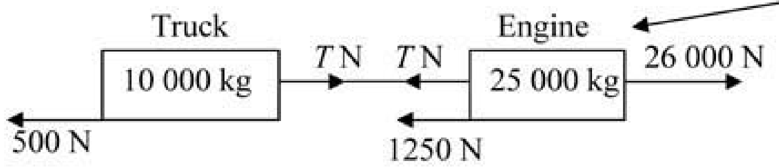
b the tension in the coupling.

The engine and the truck now climb a slope which is modelled as a plane inclined at angle α to the horizontal, where $\sin \alpha = \frac{1}{70}$.

The engine and the truck are moving up the slope with an acceleration of magnitude 0.6 m s^{-2}. The frictional resistances to motion are modelled as before.

c Calculate the tractive force exerted by the engine. Give your answer in kN. (1 tonne = 1000 kg)

Solution:



1 tonne = 1000 kg. It is recommended that you work with metres, kilograms and seconds in all questions using Newton's Second Law.

a The resistance on the engine is $25 \times 50 = 1250$ N

The resistance on the truck is $10 \times 50 = 500$ N

For the whole system, the engine and truck

$R(\rightarrow)$ $\quad$ $\mathbf{F} = m\mathbf{a}$

$$26\,000 - 1250 - 500 = 35\,000a$$

$$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928\ \ldots$$

The acceleration of the engine and truck is 0.693 m s^{-2} (3 s.f).

There is 50 N of resistance for each tonne, so an engine of 25 tonnes is resisted by a force of 50×25 N .

When you consider the whole system, the tension in the coupling acting on the truck and the tension in the coupling acting on the engine are of equal magnitude and in opposite directions. When you resolve horizontally, the tensions cancel out .

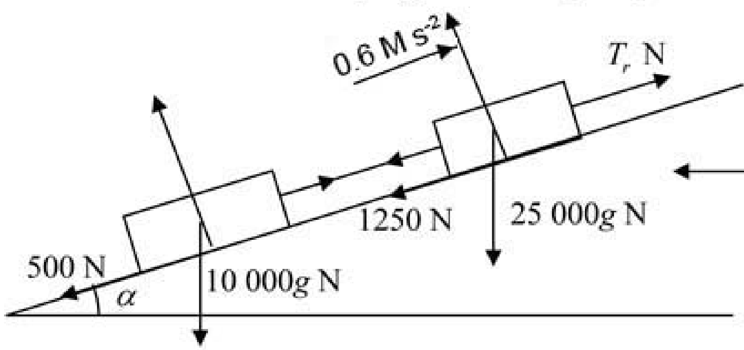
b For the truck alone

$\mathbf{F} = m\mathbf{a}$

$T - 5000 = 10\,000a$

$T = 500 + 10\,000 \times 0.6928\ldots = 7428.57\ \ldots$

The tension in the coupling is 7430 N (3 s.f.).



In this diagram, only the forces which are relevant when resolving up the plane have been labelled. The tractive force has been labelled T_r N.

c For the whole system, the engine and truck

$R(\nearrow)$

$$T_r - 2500 - 500 - 25\,000g\sin\alpha - 10\,000g\sin\alpha = 35\,000a$$

$$T_r = 1750 + 35\,000 \times 9.8 \times \frac{1}{70} + 35\,000 \times 0.6$$

$$= 27\,650$$

The tractive force of the engine is 28 kN (2 s.f.).

There are 5 forces acting parallel to the plane. The tractive force, two resistances and two components of weight.

1 kN = 1000 N

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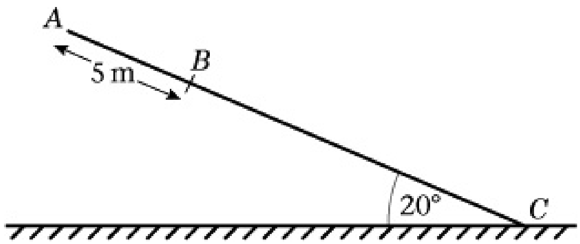
Review Exercise
Exercise A, Question 47

Question:



A suitcase of mass 10 kg slides down a ramp which is inclined at an angle of 20 ° to the horizontal. The suitcase is modelled as a particle and the ramp as a rough plane. The top of the plane is *A*. The bottom of the plane is *C* and *AC* is a line of greatest slope, as shown in the figure above. The point *B* is on *AC* with *AB* = 5 m. The suitcase leaves *A* with a speed of 10 m s^{-1} and passes *B* with a speed of 8 m s^{-1}. Find

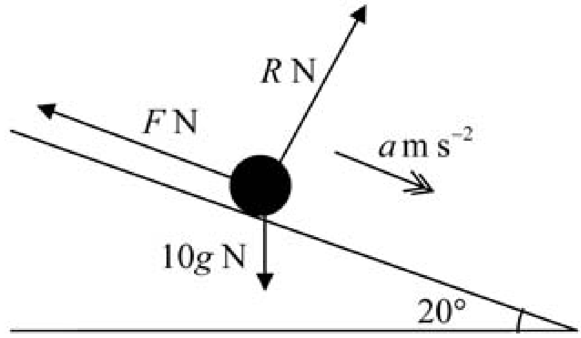
a the deceleration of the suitcase,

b the coefficient of friction between the suitcase and the ramp.

The suitcase reaches the bottom of the ramp.

c Find the greatest possible length of *AC*.

Solution:



a From A to B

$u = 10,\ v = 8,\ s = 5,\ a = ?$

$v^2 = u^2 + 2as$

$8^2 = 10^2 + 2 \times a \times 5$

$a = \dfrac{8^2 - 10^2}{10} = -3.6$

The deceleration of the suitcase is 3.6 m s^{-2}.

b $R(\nearrow)$ $\quad R - 10g\cos 20° = 0 \Rightarrow R = 10g\cos 20°$

Friction is limiting

$F = \mu R = \mu 10g\cos 20°$

$R(\searrow)$ $\quad 10g\sin 20° - F = 10a$

$\cancel{10}g\sin 20° - \mu\cancel{10}g\cos 20° = \cancel{10} \times (-3.6)$

$\mu = \dfrac{9.8\sin 20° + 3.6}{9.8\cos 20°} = 0.7548\ \ldots$

As the numerical value $g = 9.8$ has been taken, you should give your final answer for μ, corrected to 2 significant figures.

The coefficient of friction between the suitcase and the ramp is 0.75 (2 s.f.).

c From A to C

$u = 10,\ v = 0,\ a = -3.6,\ s = ?$

To reach the bottom of the ramp, the suitcase must not stop before it reaches the lowest point of the ramp C. The limiting case is that the suitcase has zero speed at C and this is taken to find the greatest possible length of AC.

$v^2 = u^2 + 2as$

$0^2 = 10^2 + 2 \times (-3.6) \times s$

$s = \dfrac{10^2}{7.2} = 13.\dot{8}$

The greatest possible length of AC is 14 m (2 s.f.).

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Review Exercise
Exercise A, Question 48

Question:

A slipway for launching boats consists of a rough straight track inclined at an angle of 10 ° to the horizontal. A boat of mass 300 kg is pulled down the slipway by means of a rope which is parallel to the slipway. When the tension in the rope is 500 N, the boat moves down the slipway with constant speed.

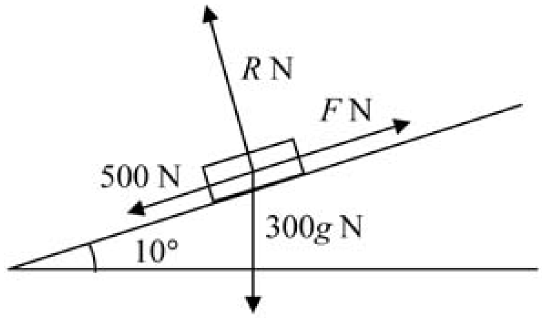
a Find, to two significant figures, the coefficient of friction between the boat and the slipway.

Later the boat returns to the slipway. It is now pulled up the slipway at constant speed by the rope which is again parallel to the slipway.

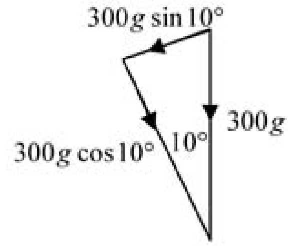
b Give a brief reason why the magnitude of the frictional force is the same as when the boat is pulled down the slope.

c Find, to two significant figures, the tension in the rope.

Solution:



The weight has components parallel and perpendicular to the plane



a $R(\nwarrow)$ $R - 300g\cos 10° = 0 \Rightarrow R = 300g\cos 10°$

Friction is limiting

$$F = \mu R = \mu 300g\cos 10°$$

$R(\swarrow)$ $500 + 300g\sin 10° - F = 300 \times 0$

The boat is moving at a constant speed and, hence, its acceleration is zero.

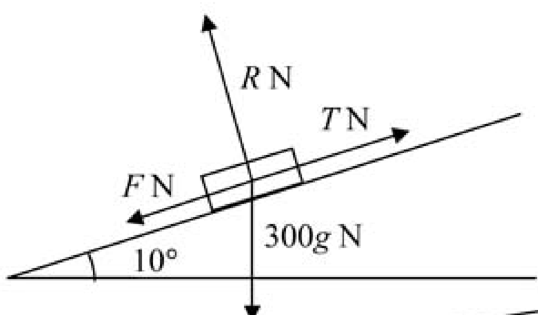
$$F = \mu 300g\cos 10° = 500 + 300g\sin 10°$$

$$\mu = \frac{500 + 300g\sin 10°}{300g\cos 10°} = 0.349\ \ldots$$

The coefficient of friction between the boat and the slipway is 0.35 (2 s.f).

b The normal reaction $(R = 300g\cos 10°)$ is unchanged and, hence, the friction force $(F = \mu R)$ is unchanged.

c



F is unchanged in magnitude but, as friction opposes motion, it has reversed direction and is now acting down the plane.

$R(\nearrow)$ $T - 300g\sin 10° - F = 300 \times 0$

$$T = 300g\sin 10° + F$$
$$= 600g\sin 10° + 500$$
$$= 1521\ \ldots$$

$F = 500 + 300g\sin 10°$ from the working in part **a**.

The tension in the rope is 1500 N (2 s.f.).

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Review Exercise
Exercise A, Question 49

Question:

A tent peg is driven into soft ground by a blow from a hammer. The tent peg has mass 0.2 kg and the hammer has mass 3 kg. The hammer strikes the peg vertically.

Immediately before the impact, the speed of the hammer is 16 m s^{-1}. It is assumed that, immediately after the impact, the hammer and the peg move together vertically downwards.

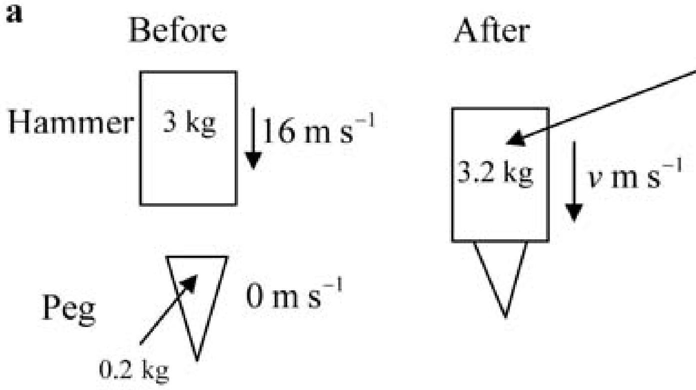
a Find the common speed of the peg and the hammer immediately after the impact.

Until the peg and hammer come to rest, the resistance exerted by the ground is assumed to be constant and of magnitude *R* newtons. The hammer and peg are brought to rest 0.05 s after the impact.

b Find, to three significant figures, the value of *R*.

Solution:

a



After impact, the hammer and the peg move together. You model the hammer and peg as a single particle of mass 3.2 kg

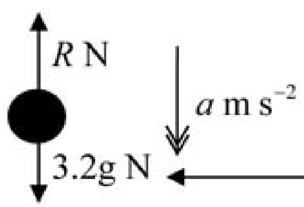
Conservation of linear momentum.

$$3\times 16+0.2\times 0=3.2\times v$$

$$v=\frac{3\times 16}{3.2}=15$$

The common speed of the peg and the hammer immediately after the impact is 15 m s^{-1}.

b



$u=15,\ v=0,\ t=0.05,\ a=?$

$v=u+at$

$0=15+a\times 0.05\ \Rightarrow\ a=-\dfrac{15}{0.05}=-300$

$\mathbf{F}=m\mathbf{a}$

$3.2g-R=3.2\times(-300)$

$R=3.2\times 300+3.2\times 9.8=991.36$

$\quad =991\ (3\text{ s.f.})$

You need to find R using $\mathbf{F}=m\mathbf{a}$. Before this can be used, you will need to find the deceleration of the hammer and the peg. As in part **a**, you need to consider the hammer and the peg as a single particle of mass 3.2 kg.

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Review Exercise
Exercise A, Question 50

Question:

A ball is projected vertically upwards with a speed u m s^{-1} from a point A which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

a Show that $u = 22.4$.

The ball reaches the ground T seconds after it has been projected from A.

b Find, to two decimal places, the value of T.

The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

c Find, to three significant figures, the value of F.

d State one physical factor which could be taken into account to make the model used in this question more realistic.

Solution:

a From A to the greatest height, taking upwards as positive.

$v = 0, a = -9.8, s = 25.6, u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

The ball reaches the ground at a point which is 1.5 m lower than the point of projection A. So you must take $s = -1.5$.

b $u = 22.4, s = -1.5, a = -9.8, t = T$

$s = ut + \frac{1}{2}at^2$

$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

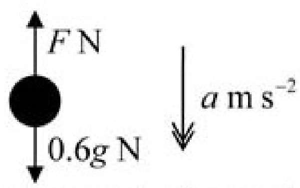
$4.9T^2 - 22.4T - 1.5 = 0$

You can ignore the negative solution of the quadratic equation. That would represent a time before the ball was projected.

$T = \dfrac{22.4 + \sqrt{(22.4^2 - 4 \times 4.9 \times -1.5)}}{2 \times 9.8}$

$= 4.637 \ldots = 4.64$ (3 s.f.).

c



To find the speed of the ball as it reaches the ground.

$u = 22.4, s = -1.5, a = -9.8, v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

As, at the next stage, you will use the velocity squared, you need not find the square root of 531.16. The final velocity of the motion under gravity becomes the initial velocity of the motion as the ball sinks into the ground.

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16, v = 0, s = 0.025, a = ?$

You need to use metres, kilograms and seconds consistently, so 2.5 cm must be converted to 0.025 m.

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

$a = -\dfrac{531.16}{0.05} = -10623.2$

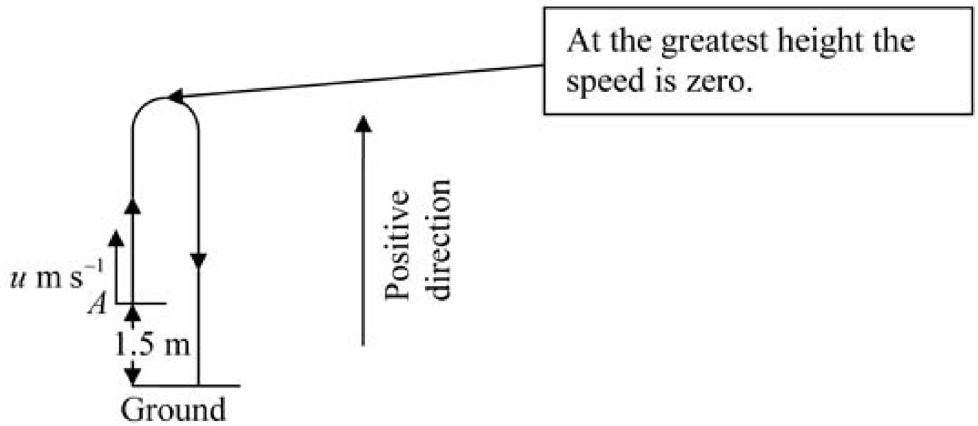
$\mathbf{F} = m\mathbf{a}$

$0.6g - F = 0.6 \times (-10\,623.2)$

$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.).

d Consider air resistance during motion under gravity.

To use a variable F, as resisting forces usually vary with speed, would also be a good answer.


At the greatest height the speed is zero.
Positive direction
u m s^{-1}
A
1.5 m
Ground

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 51

Question:

A particle *A*, of mass 0.8 kg, resting on a smooth horizontal table, is connected to a particle *B*, of mass 0.6 kg, which is 1 m from the ground, by a light inextensible string passing over a small pulley at the edge of the table. The particle *A* is more than 1 m from the edge of the table. The system is released from rest with the horizontal part of the string perpendicular to the edge of the table, the hanging parts vertical and the string taut. Calculate

a the acceleration of *A*,

b the tension in the string,

c the speed of *B* when it hits the ground,

c the time taken for *B* to reach the ground.

Solution:

a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

> The ball reaches the ground at a point which is 1.5 m lower than the point of projection A. So you must take $s = -1.5$.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

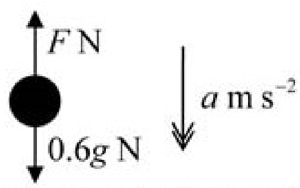
$4.9T^2 - 22.4T - 1.5 = 0$

> You can ignore the negative solution of the quadratic equation. That would represent a time before the ball was projected.

$$T = \frac{22.4 + \sqrt{(22.4^2 - 4 \times 4.9 \times -1.5)}}{2 \times 9.8}$$

$= 4.637\ \ldots = 4.64$ (3 s.f.).

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

> As, at the next stage, you will use the velocity squared, you need not find the square root of 531.16. The final velocity of the motion under gravity becomes the initial velocity of the motion as the ball sinks into the ground.

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

> You need to use metres, kilograms and seconds consistently, so 2.5 cm must be converted to 0.025 m.

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

$a = -\frac{531.16}{0.05} = -10623.2$

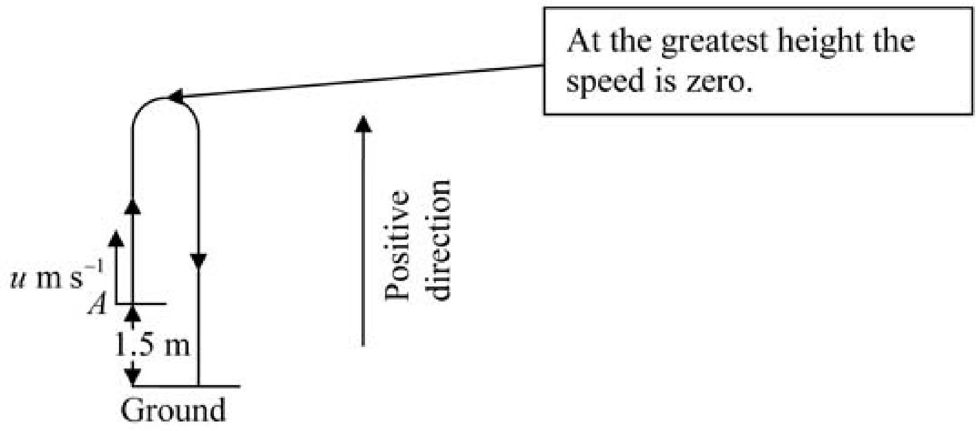
$\mathbf{F} = m\mathbf{a}$

$0.6g - F = 0.6 \times (-10\,623.2)$

$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.).

d Consider air resistance during motion under gravity.

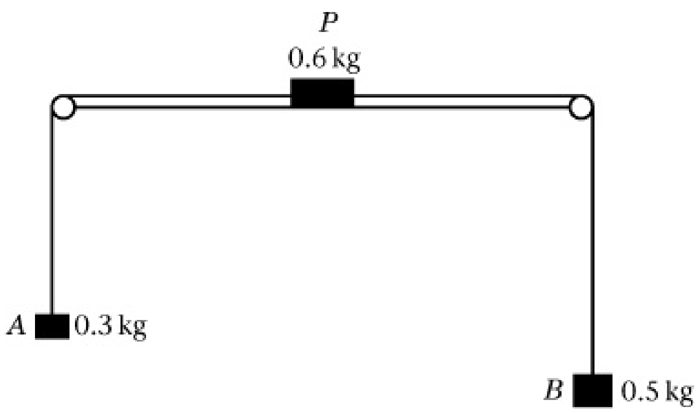
> To use a variable F, as resisting forces usually vary with speed, would also be a good answer.


At the greatest height the speed is zero.
Positive direction
u m s^{-1}
A
1.5 m
Ground

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 52

Question:

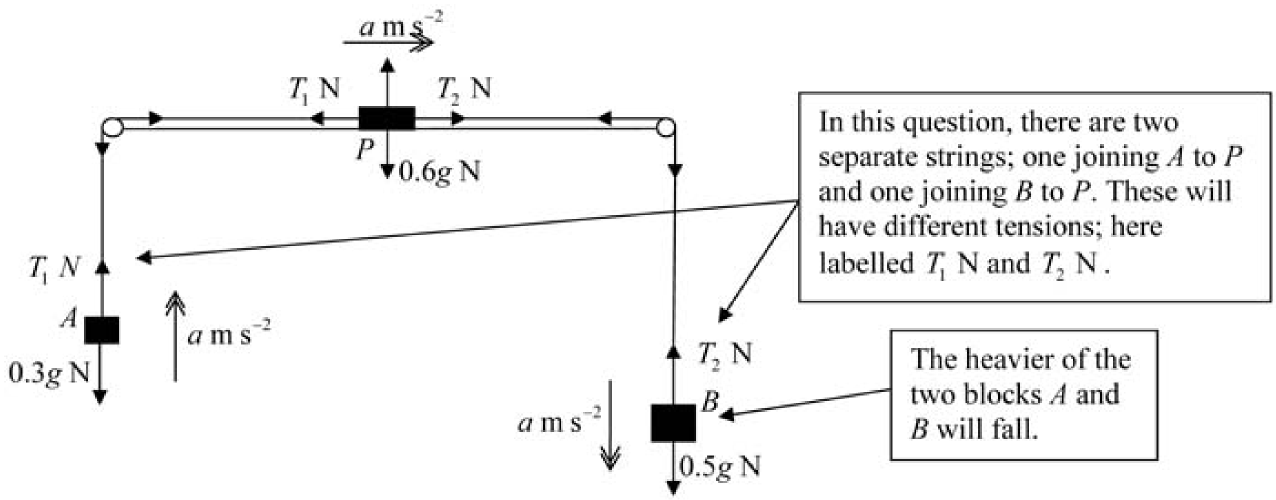


The figure shows a block P of mass 0.6 kg resting on the smooth surface of a horizontal table. Inextensible light strings connect P to blocks A and B which hang freely over light smooth pulleys placed at opposite parallel edges of the table. The masses of A and B are 0.3 kg and 0.5 kg respectively. All portions of the string are taut and perpendicular to their respective edges of the table. The system is released from rest. Calculate

a the common magnitude of the accelerations of the blocks,

b the tensions in the strings.

Solution:



a For A

$R(\uparrow)$ $\quad T_1 - 0.3g = 0.3a \ \dots\dots (1)$

For P

$R(\rightarrow)$ $\quad T_2 - T_1 = 0.6a \ \dots\dots (2)$

For B

$R(\downarrow)$ $\quad 0.5g - T_2 = 0.5a \ \dots\dots (3)$

(1) + (2) + (3)

$$0.2g = 1.4a$$

$$a = \frac{0.2 \times 9.8}{1.4} = 1.4$$

The common magnitude of the accelerations of the blocks is 1.4 m s^{-2}.

3 equations in 3 unknowns can be difficult to solve but, in this case, if you add the 3 equations together the T_1 in (1) cancels with the $-T_1$ in (2) and the T_2 in (2) cancels out with the $-T_2$ in (3), leaving an equation in a alone.

b Substitute the result of part **a** into (1)

$$T_1 - 0.3g = 0.3 \times 1.4$$

$$T_1 = 0.3 \times 1.4 + 0.3 \times 9.8 = 3.36 \ \dots (4)$$

Substitute the result of part **a** and (4) into (2)

$$T_2 - 3.36 = 0.6 \times 1.4$$

$$T_2 = 3.36 + 0.6 \times 1.4 = 4.2$$

Alternatively, you can find T_2 by substituting for a in (3).

The tension in the string joining A to P is 3.4 N (2 s.f.).
The tension in the string joining B to P is 4.2 N.

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Review Exercise
Exercise A, Question 53

Question:

A trailer of mass 600 kg is attached to a car of mass 900 kg by means of a light inextensible tow-bar. The car tows the trailer along a horizontal road. The resistances to motion of the car and trailer are 300 N and 150 N respectively.

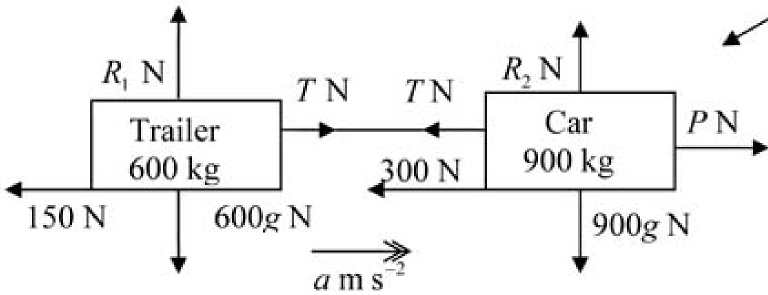
a Given that the acceleration of the car and trailer is 0.4 m s^{-2}, calculate

i the tractive force exerted by the engine of the car,

ii the tension in the tow bar.

b Given that the magnitude of the force in the tow-bar must not exceed 1650N, calculate the greatest possible deceleration of the car.

Solution:



The tractive force exerted by the engine of the car has been called P N. This only acts on the car. It does not act directly on the trailer. The only force moving the trailer forward is the tension in the tow bar.

a(i) For the whole system

$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \quad P - 300 - 150 = 1500 \times 0.4$$

$$P = 1050$$

The tractive force exerted by the engine of the car is 1050 N.

When you consider the whole system, the tension in the tow bar acting on the truck and the tension in the tow bar acting on the engine are of equal magnitude and in opposite directions. When you resolve horizontally, the tensions cancel out .

(ii)For the trailer alone

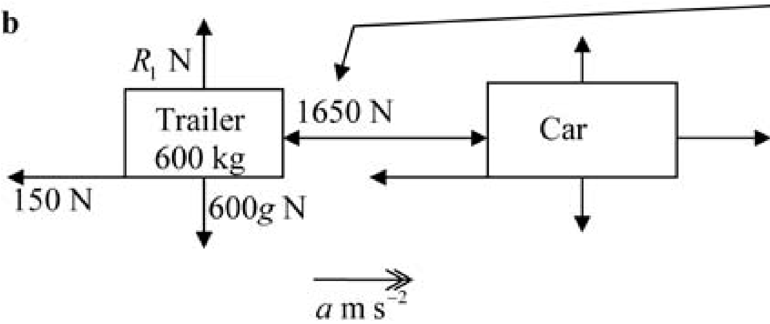
$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \qquad T - 150 = 600 \times 0.4$$

$$T = 390$$

The tension in the tow bar is 390 N.

b



When decelerating the force in the tow bar becomes a thrust. The question gives that the greatest magnitude of the thrust is 1650 N. To solve part **b**, you need only the horizontal forces on the trailer.

For the trailer alone

$$\mathbf{F} = m\mathbf{a}$$

$$R(\rightarrow) \qquad -1650 - 150 = 600a$$

$$a = -\frac{1800}{600} = -3$$

The greatest possible deceleration of the car is 3 m s^{-2}.

The deceleration of the trailer and the car are the same. Although the question asks for the deceleration of the car, you could not directly find this, as it would involve an unknown braking force on the car.

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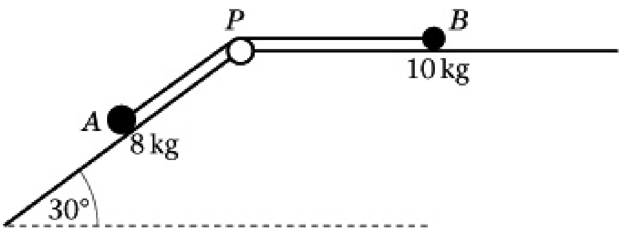
Review Exercise
Exercise A, Question 54

Question:



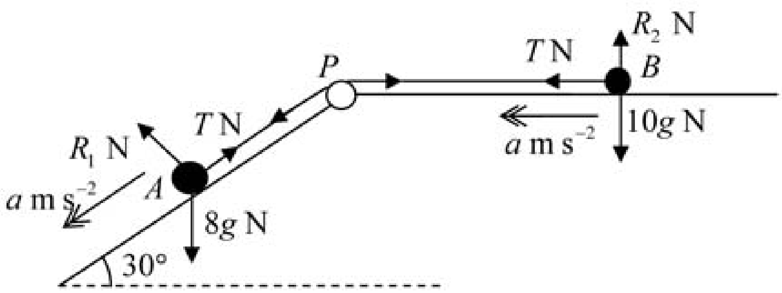
Two particles *A* and *B*, of mass 8 kg and 10 kg respectively, are connected by a light inextensible string which passes over a light smooth pulley *P*. Particle *B* rests on a smooth horizontal table and *A* rests on a smooth plane inclined at 30 ° to the horizontal with the string taut and perpendicular to the line of intersection of the table and the plane as shown in the figure. The system is released from rest. Find

a the magnitude of the acceleration of *B*,

b the tension in the string,

c the distance covered by *B* in the first two seconds of motion, given that *B* does not reach the pulley.

Solution:



a For B

$R(\leftarrow)$ $\quad T = 10a \quad \ldots \ldots (1)$

For A

$R(\swarrow)$ $\quad 8g\sin 30° - T = 8a \quad \ldots \ldots (2)$

(1) + (2)

$$8g\sin 30° = 18a$$

$$a = \frac{8g\sin 30°}{18} = \frac{2}{9}g = 2.1\dot{7}$$

The magnitude of the acceleration of B is 2.2 m s^{-2} (2 s.f.).

(1) and (2) are a pair of linear simultaneous equations in T and a. The methods you can use to solve these is essentially the same as you learnt for GCSE. Either elimination or substitution can be used.

$$\sin 30° = \frac{1}{2}$$

b Substitute $a = \frac{2}{9}g$ into (1)

$$T = \frac{20}{9}g = 21.\dot{7}$$

The tension in the string is 22 N (2 s.f.).

c $u = 0, a = \frac{2}{9}g, t = 2, s = ?$

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{g}{9} \times 4 = \frac{9.8 \times 4}{9} = 4.3\dot{5}$$

The distance covered in the first 2 seconds is 4.4 m (2 s.f.).

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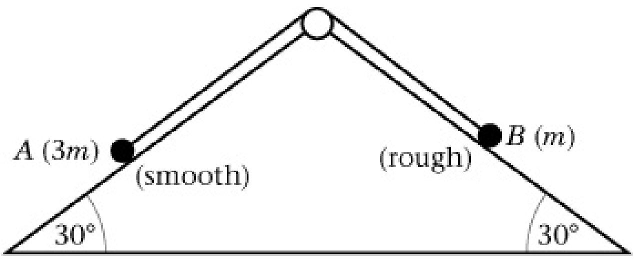
Review Exercise
Exercise A, Question 55

Question:



A fixed wedge has two plane faces, each inclined at 30 ° to the horizontal. Two particles *A* and *B*, of mass 3*m* and *m* respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a smooth light pulley fixed at the top of the wedge. The face on which *A* moves is smooth. The face on which *B* moves is rough. The coefficient of friction between *B* and this face is μ. Particle *A* is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in the figure.

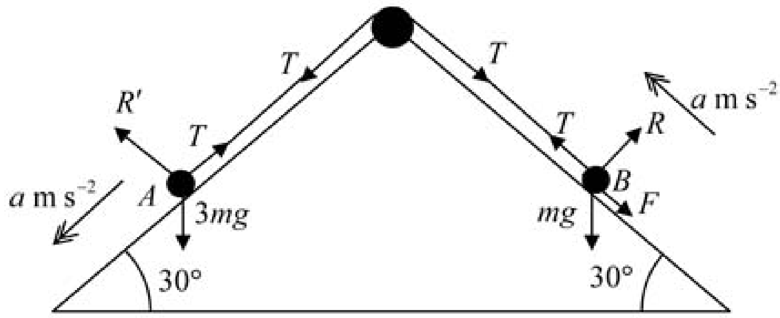
The particles are released from rest and start to move. Particle *A* moves downwards and particle *B* moves upwards. The acceleration of *A* and *B* each have magnitude $\frac{1}{10}g$.

a By considering the motion of *A*, find, in terms of *m* and *g*, the tension in the string.

b By considering the motion of *B*, find the value of μ.

c Find the resultant force exerted by the string on the pulley, giving its magnitude and direction.

Solution:



a For A $\qquad \mathbf{F} = m\mathbf{a}$

$R(\swarrow)\ 3mg\sin 30° - T = 3m \times \frac{1}{10} g$

$T = \frac{3}{2}mg - \frac{3}{10}mg = \frac{6}{5}mg$

The tension in the string is $\frac{6}{5}mg$.

$\sin 30° = \frac{1}{2}$

b For B

$R(\nearrow)\quad R - mg\cos 30° = 0 \Rightarrow R = mg\cos 30°$

Friction is limiting

$F = \mu R = \mu mg\cos 30°$

$R(\nwarrow)\quad T - F - mg\sin 30° = ma$

$\frac{6}{5}\cancel{mg} - \mu\cancel{mg}\cos 30° - \cancel{mg}\sin 30° = \frac{1}{10}\cancel{mg}$

$\mu\cos 30° = \frac{6}{5} - \frac{1}{2} - \frac{1}{10} = \frac{3}{5}$

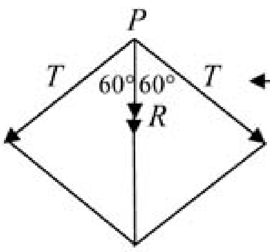
$\mu = \dfrac{3}{5\cos 30°} = 0.692\,82\ \ldots$

$= 0.693$ (3 s.f.).

mg is common to all 4 terms in this equation and can be 'cancelled' leaving an equation in μ alone.

$\cos 30° = \dfrac{\sqrt{3}}{2}$ is the exact value and the exact answer for $\mu\left(= \dfrac{2\sqrt{3}}{5}\right)$ would be acceptable.

c



At P

$R(\downarrow)\quad R = 2T\cos 60°$

$= 2 \times \frac{6}{5}mg \times \frac{1}{2} = \frac{6}{5}mg$

The resultant force exerted by the string on the pulley has magnitude $\frac{6}{5}mg$ and acts vertically downwards.

At P, the string exerts two forces, each of magnitude $\frac{6}{5}mg$, on the pulley both at an angle of $60°$ to the vertical. By symmetry, the resultant force acts vertically downwards. Its magnitude can be found by resolving the two forces vertically downwards.

$\cos 60° = \frac{1}{2}$ is the exact value.

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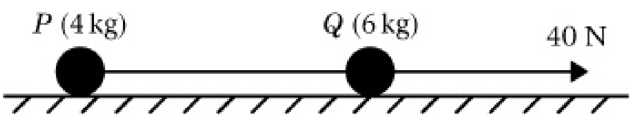
Review Exercise
Exercise A, Question 56

Question:



Two particles *P* and *Q*, of mass 4 kg and 6 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. The coefficient of friction between each particle and the plane is $\frac{2}{7}$. A constant force of magnitude 40 N is then applied to *Q* in the direction *PQ*, as shown in the figure.

a Show that the acceleration of *Q* is 1.2 m s^{-2}.

b Calculate the tension in the string when the system is moving.

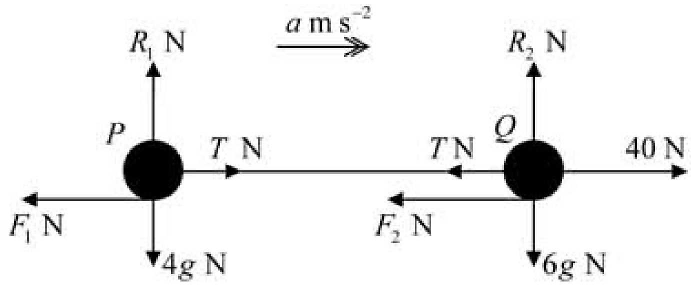
c State how you have used the information that the string is inextensible.

After the particles have been moving for 7 s, the string breaks. The particle *Q* remains under the action of the force of magnitude 40N.

d Show that *P* continues to move for a further 3 seconds.

e Calculate the speed of *Q* at the instant when *P* comes to rest.

Solution:



a For P, $R(\uparrow)$ $R_1 - 4g = 0 \Rightarrow R_1 = 4g$

Friction is limiting

$$F_1 = \mu R_1 = \tfrac{2}{7} \times 4g = \tfrac{8}{7}g$$

For Q, $R(\uparrow)$ $R_2 - 6g = 0 \Rightarrow R_2 = 6g$

Friction is limiting

$$F_2 = \mu R_2 = \tfrac{2}{7} \times 6g = \tfrac{12}{7}g$$

For the whole system

$R(\rightarrow)$ $\quad 40 - F_1 - F_2 = 10a$

$$40 - \tfrac{8}{7}g - \tfrac{12}{7}g = 10a$$

$$10a = 40 - \tfrac{20}{7} \times 9.8 = 12 \Rightarrow a = 1.2$$

The acceleration of Q is 1.2 m s^{-2}, as required.

> When a question asks you to show that a quantity has a value – here that the acceleration is 1.2 m s^{-1} - you must get exactly that value and not approximate during the calculation.

b For P

$R(\rightarrow)$ $\quad T - F_1 = 4a$

$$T - \tfrac{8}{7}g = 4 \times 1.2$$

$$T = 4 \times 1.2 + \tfrac{8}{7} \times 9.8 = 16$$

The tension in the string is 16 N.

c The information that the string is inextensible has been used in assuming that the accelerations of P and Q, and hence of the whole system, are the same.

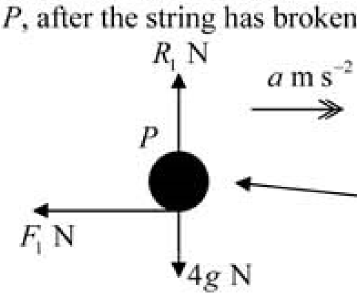
d To find the speed the particles are travelling at when the string breaks.

$u = 0, a = 1.2, t = 7, v = ?$

$v = u + at = 0 + 1.2 \times 7 = 8.4$

> The final speed for the part of the motion when the string is taut will be the initial speed of both particles after the string breaks.

For P, after the string has broken



> After the string has broken it no longer exerts a tension on P. The forces acting on P are shown in the diagram. The equation obtained by resolving vertically is unchanged and so the normal reaction and the friction force at P are unchanged.

$R(\rightarrow) \quad -F_1 = 4a \Rightarrow -\frac{8}{7}g = 4a \Rightarrow a = -\frac{2}{7}g$

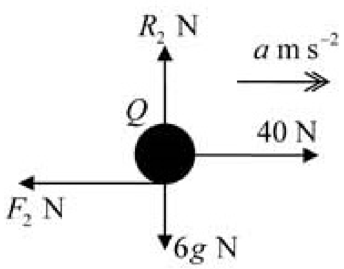
$u = 8.4, v = 0, a = -\frac{2}{7}g, t = ?$

$v = u + at$

$0 = 8.4 - \frac{2}{7}gt \Rightarrow t = \dfrac{8.4 \times 7}{2 \times 9.8} = 3$

P continues to move for a further 3 s, as required.

e



After the string has broken it no longer exerts a tension on Q. The forces acting on Q are shown in the diagram. The equation obtained by resolving vertically is unchanged and so the normal reaction and the friction force at Q are unchanged.

For Q, after the string has broken.

$R(\rightarrow) \quad 40 - F_2 = 6a$

$40 - \frac{12}{7}g = 6a$

$6a = 40 - \frac{12}{7} \times 9.8 = 23.2$

$a = \dfrac{23.2}{6} = \dfrac{58}{15} = 3.8\dot{6}$

$u = 8.4, a = \dfrac{58}{15}, t = 3, v = ?$

$v = u + at = 8.4 + \dfrac{58}{15} \times 3 = 20$

P came to rest 3 seconds after the string had broken. So you have been asked to find the speed of Q after these 3 seconds. First you need to find acceleration of Q. As P is not now attached to Q, Q will accelerate more quickly.

The speed of Q at the instant when P comes to rest is 20 m s^{-1}.

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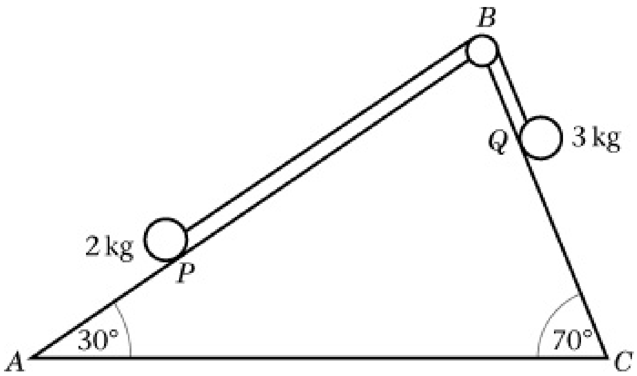
Review Exercise
Exercise A, Question 57

Question:



A fixed wedge whose smooth faces are inclined at 30° and 70° to the horizontal has a small smooth pulley fixed on the top edge at *B*. A light inextensible string, passing over the pulley, has particles *P* and *Q* of mass 2 kg and 3 kg respectively attached at its ends. The figure shows a vertical cross-section of the wedge where *AB* and *AC* are lines of greatest slope of the faces along which *P* and *Q* respectively can slide. The particles are released from rest at time $t = 0$ with the string taut. Assuming that *P* has not reached *B* and that *Q* has not reached *C*, find

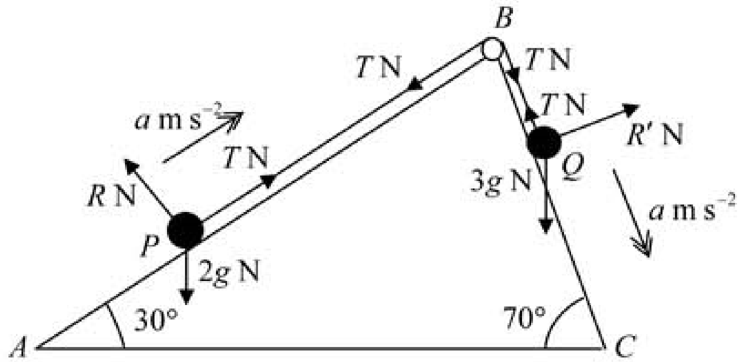
a the distance through which each particle has moved when $t = 0.75$ s,

b the tension in the string,

c the magnitude and direction of the resultant force exerted on the pulley by the string.

When $t = 0.75$ s the string breaks and in the subsequent motion P come to instantaneous rest at time t_1. Assuming that *P* has not reached *B*, **d** calculate *t*.

Solution:



a For P

$R(\nearrow)$ $T - 2g\sin 30° = 2a$ … … (1)

For Q

$R(\searrow)$ $3g\sin 70° - T = 3a$ … … (2)

(1) + (2)

$3g\sin 70° - 2g\sin 30° = 5a$

$a = \dfrac{3g\sin 70° - 2g\sin 30°}{5} = 3.565\,392\ldots$

$u = 0, a = 3.565\ldots, t = 0.75, s = ?$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 3.565\ldots \times 0.75^2 = 1.002\ldots$

The distance the particles have moved is 1.0 m (2 s.f.).

(1) and (2) are a pair of linear simultaneous equations. You need to solve them for a to solve part **a**. You need to solve them for T in part **b**.

You need the value of a several times in the question and it is sensible to store this value of a in your calculator so you can recall it when needed.

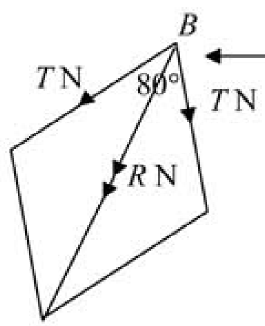
b From (1)

$T = 2g\sin 30° + 2a$

$= 2g\sin 30° + 2 \times 3.565\ldots = 16.93\ldots$

The tension in the string is 17 N (2 s.f.).

c



At B, the string exerts two forces, each of magnitude T. The resultant force bisects the angle ABC, which is $80°$. Its magnitude can be found by resolving the two forces along the diagonal of the rhombus.

$R = 2T\cos 40° = 2 \times 16.93\ldots \times \cos 40° = 25.939\ldots$

The resultant force exerted on the pulley by the string has magnitude 26 N (2 s.f.), and acts in the direction bisecting $\angle ABC$, as shown in the diagram above.

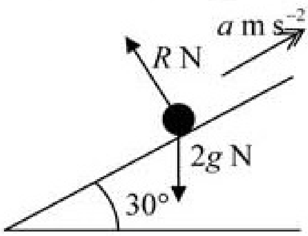
d To find the speed of P immediately before the string breaks

$u = 0, t = 0.75, a = 3.565\ \dots, v = ?$

$v = u + at = 0 + 0.75 \times 3.565 = 2.674\ \dots$

The final speed for the part of the motion when the string is taut will be the initial speed of P after the string breaks.

For P, after the string breaks



$R(\nearrow) \quad -2g\sin 30° = 2a$

$a = -g\sin 30° = -\frac{1}{2}g$

After the string has broken, the only force acting on P in the direction parallel to the plane is the component of the weight acting down the plane.

$u = 2.674\ \dots, v = 0, a = -\frac{1}{2}g, t = t_1$

$v = u + at$

$0 = 2.674\ \dots - \frac{1}{2}gt_1$

$\sin 30° = \frac{1}{2}$

$t_1 = \frac{2 \times 2.674\ \dots}{9.8} = 0.545\,7\ \dots$

$= 0.55$ (2 s.f.).

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Review Exercise
Exercise A, Question 58

Question:

A car of total mass 1200 kg is moving along a straight horizontal road at a speed of 40 m s^{-1}, when the driver makes an emergency stop. When the brakes are fully applied, they exert a constant force and the car comes to rest after travelling a distance of 80 m. The resistance to motion from all factors other than the brakes is assumed to be constant and of magnitude 500 N.

a Find the magnitude of the force by the brakes when fully applied.

A trailer, with no brakes, is now attached to the car by means of a tow-bar. The mass of the trailer is 600 kg, and when the trailer is moving, it experiences a constant resistance to motion of magnitude 420 N. The tow-bar may be assumed to be a light rigid rod which remains parallel to the road during motion. The car and the trailer come to a straight hill, inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. They move together down the hill. The driver again makes an emergency stop, the brakes applying the same force as when the car was moving along level ground.

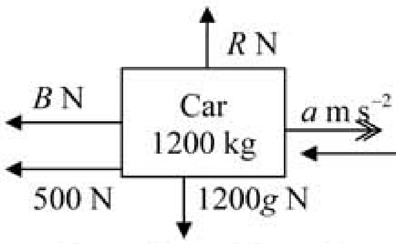
b Find the deceleration of the car and the trailer when the brakes are fully applied.

c Find the magnitude of the force exerted on the car by the trailer when the brakes are fully applied.

d Find the maximum speed at which the car and trailer should travel down the hill to ensure that, when the brakes are fully applied, they can stop within 80m.

Solution:

a



The force exerted by the brakes is labelled as B N in this diagram. During braking both the braking force and the resistance act to slow the car down.

$u = 40$, $v = 0$, $s = 80$, $a = ?$

$v^2 = u^2 + 2as$

$0^2 = 40^2 + 2 \times a \times 80$

$a = -\dfrac{40^2}{2 \times 80} = -10$

To find the braking force, you will have to use $\mathbf{F} = m\mathbf{a}$ and so the first step in the question is to find the deceleration.

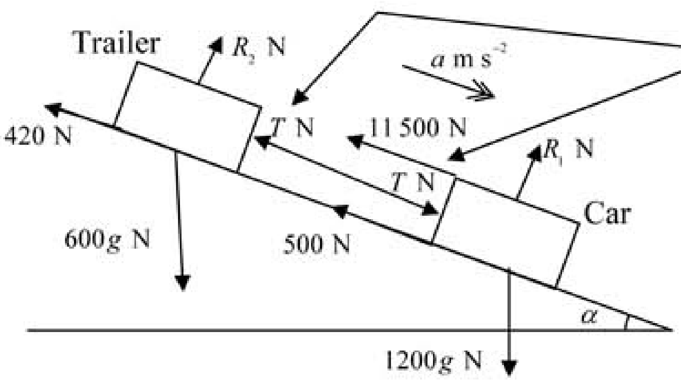
$$\mathbf{F} = m\mathbf{a}$$

$R(\rightarrow)\quad -B - 500 = 1200 \times (-10)$

$B = 12\,000 - 500 = 11\,500$

The magnitude of the force exerted by the brakes is 11 500 N.

b



Under braking, the force in the tow bar is a thrust and acts outwards in the directions shown in the diagram. When Newton's second law is applied to the whole system, the forces at the ends of the tow bar are of equal magnitude and in opposite directions and so they cancel one another out.

For the car and the trailer

$R(\square)$

$$\underbrace{1200g\sin\alpha + 600g\sin\alpha}_{\text{Components of weight}} - \underbrace{500 - 420}_{\text{Resistances}} - \underset{\substack{\uparrow \\ \text{Braking force}}}{11\,500} = \underset{\substack{\nwarrow \\ \text{Total Mass}}}{1800}a$$

$1800 \times 9.8 \times \dfrac{1}{14} - 12\,420 = 1800a$

$1260 - 12\,420 = 1800a$

$a = -\dfrac{11160}{1800} = -6.2$

The deceleration of the car and the trailer is $6.2\ \text{m s}^{-2}$.

You are given that $\sin\alpha = \dfrac{1}{14}$ and you should use that exact value here. It is not necessary to use your calculator to find a value for α. Any such value would only be approximate and would lead to inaccuracy.

c For the car alone

$R(\square\ )$

$T + 1200g\sin\alpha - 500 - 11\,500 = 1200a$

$T = 1200\times(-6.2) - 1200\times 9.8\times\frac{1}{14} + 500 + 11\,500$

$= 3720$

The magnitude of the force exerted on the car by the trailer is 3700 N (2 s.f.).

The trailer exerts a force on the car through the thrust in the tow bar. That thrust acts down the plane in the same direction as the component of the weight. The braking force and the resistance act up the plane

d $a = -6.2,\ v = 0,\ s = 80,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2\times(-6.2)\times 80$

$u^2 = 2\times 6.2\times 80 = 992$

$u = \sqrt{992} = 31.496\ \ldots$

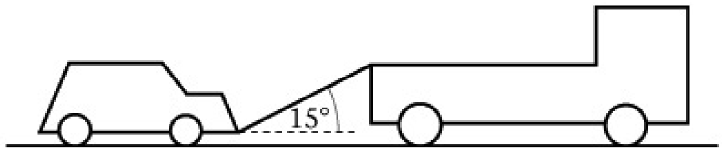
The maximum speed at which the car and trailer should travel down the hill to ensure that, when the brakes are fully applied, they can stop within 80 m is 31 m s^{-1} (2 s.f.).

If the car and trailer were travelling at a slower speed, they could stop in less than 80 m.

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

Review Exercise
Exercise A, Question 59

Question:



The figure above shows a lorry of mass 1600 kg towing a car of mass 900 kg along a straight horizontal road. The two vehicles are joined by a light tow-bar which is at an angle of $15°$ to the road. The lorry and the car experience constant resistances to motion of magnitude 600 N and 300 N respectively. The lorry's engine produces a constant horizontal force on the lorry of magnitude 1500 N. Find

a the acceleration of the lorry and the car,

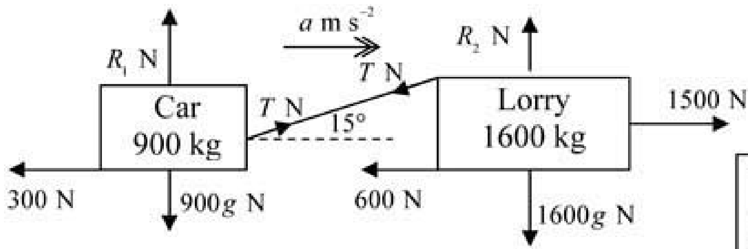
b the tension in the tow-bar.

When the speed of the vehicles is 6 m s^{-1}, the tow-bar breaks. Assuming that the resistance to the motion of the car remains of constant magnitude 300 N,

c find the distance moved by the car from the moment the tow-bar breaks to the moment when the car comes to rest.

d State whether, when the tow-bar breaks, the normal reaction of the road on the car is increased, decreased or remains constant. Give a reason for your answer.

Solution:



When you consider the whole system, the tension in the tow bar acting on the car and tension in the tow bar acting on the lorry are of equal magnitude and in opposite directions. When you resolve in any direction for the whole system, the tensions cancel each other out .

a For the lorry and the car

$R(\rightarrow)$ $\quad 1500 - 600 - 300 = 2500a$

$$a = \frac{600}{2500} = 0.24$$

The acceleration of the lorry and the car is 0.24 m s^{-2}.

b For the car alone

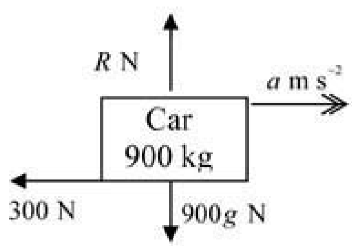
$$\mathbf{F} = m\mathbf{a}$$

$R(\rightarrow)$ $\quad T\cos 15° - 300 = 900 \times 0.24$

$$T = \frac{900 \times 0.24 + 300}{\cos 15°} = 534.20\ \ldots$$

The tension in the tow bar is 530 N (2 s.f.).

c



After the tow bar breaks, the only force acting on the car in the horizontal direction is the constant resistance to motion.

After the tow bar breakes, for the car

$R(\rightarrow)$ $\quad -300 = 900a \Rightarrow a = -\frac{1}{3}$

$u = 6, v = 0, a = -\frac{1}{3}, s = ?$

$v^2 = u^2 + 2as$

$0^2 = 6^2 - 2 \times \frac{1}{3} \times s$

$$s = \frac{3}{2} \times 36 = 54$$

The distance moved by the car from the moment the tow bar breaks to the moment when the car comes to rest is 54 m.

d After the tow bar has broken, in part **c**, $R(\uparrow)$

$$R = 900g.$$

Before the tow bar has broken, in part **a**,
$R(\uparrow)$ for car

$$R_1 + T\sin 15° - 900g = 0 \Rightarrow R_1 = 900g - T\sin 15° < 900g$$

So the normal reaction of the road on the car is increased when the tow bar breaks.

It is a common error to omit the vertical component of the tension here. That would lead to the incorrect conclusion that the normal reaction is unchanged.